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OPTIMAL PORTFOLIO CHOICE WITH ABSORBING MARKOV CHAINS: APPLICATION TO MARKETS THAT MAY POTENTIALLY DECOUPLE

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We develop a model of optimal asset allocation with a market that has the potential to decouple. There are three Markov regimes: a regime where the market remains fully investable, a second regime where the market may become potentially decouple, and a third regime where the market becomes decoupled and investors lose all capital. The investor wishes to hold the potentially decoupled market as it can provide a source of returns that can be partially liquidated to provide intermediate consumption. With the framework, we compute certainty equivalents of foregoing investment in the potentially decoupling market and investigate a range of comparative statics including varying the probability of decoupling.

1 Introduction

Markets sometimes decouple from the world finance system, and during those periods of segmentation, investors can lose partial or all access to their capital. In March 2022, Russia was declared "uninvestable" by major index providers after Russia's links with global markets were severed, its foreign reserves frozen, and major Western countries imposed sanctions as a consequence for Russia invading Ukraine in February 2022. In particular, FTSE Russell declared Russia an "unclassified market" and removed it from its indexes on March 7, 2022, and MSCI classified Russia as a "standalone market" and removed Russia from the MSCI Emerging Markets Index (EM) on March 9, 2022.[1] When the Russian market decoupled, investors effectively lost access to their Russian investments at close to zero value. Although the example of Russia decoupling is recent, Dimson *et al.* (2023) report that the phenomenon of markets—some of them very large—becoming uninvestable has periodically occurred throughout the history of capital markets: at the start of the 20th century, Austria, Russia, and China were markets that ceased to exist over the next few decades.[2]

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In this paper, we examine how the possibility of a market becoming decoupled can affect optimal asset allocation. We model the event of decoupling as an absorbing state in a Markov chain—conditional on the market changing status to decoupled, we assume investors lose their capital in the decoupled market. Before that event happens, investors may optimally allocate to the market that has the possibility of decoupling because it currently provides a potential return stream or a source of diversification. In our model, an investor receives utility over a consumption stream, which is paid out of the investment portfolio similar to the spending rule of an endowment or a retired person. Investors can partially fund their consumption by liquidating positions in that market so long as it does not transition to becoming decoupled.

The model is constructed as follows. For simplicity, we work with two assets: a market that may decouple, which we take in hypothetical exercises to represent an emerging market country index return, and we take MSCI Emerging Market Index as the second asset. The investor rebalances at the monthly frequency and consumes a constant fraction of their portfolio each month. We vary the investor's horizon but concentrate on 1-, 5-, and 10-year horizons. We specify three Markov states (or regimes): a "Fully Investable" state where there is no probability of the market decoupling over the next month, a "Potentially Decouple" state where the market can transition to becoming decoupled or transition back to the first Fully Investable state, and the third "Decoupled" regime where the market experiences a -100% return and only the Emerging Market Index is available as an investment going forward. The Decoupled regime is modeled as an absorbing state of the Markov chain.

The probability of a market decoupling is constant transitioning from the second Potentially Decouple regime to the third Decoupled regime, but the first Fully Investable and second Potentially Decouple regimes are persistent, following Hamilton (1989). This introduces time-varying transition probabilities and predictability—conditional on being in the Fully Investable regime, for example, the system is more likely to remain in the Fully Investable regime next period than transition to the Potentially Decouple regime, and in the Fully Investable regime, there is no probability of becoming decoupled over the next period. In the Potentially Decouple regime, we are more likely to remain in that regime or transition back to the Fully Investable regime as the probability of decoupling over the next month is small. As Merton (1971) and others note, this time-varying predictability induces hedging demands, where there is a difference between the investor's one-period ahead and long-horizon optimal holdings. In the hypothetical calibrations, we show that hedging demands, although relatively small, can be positive with a 5% probability of becoming decoupled each year.

We compute certainty equivalents (or willingness-to-pay) of *not* being able to invest in a market that may transition to being decoupled, following Kandel and Stambaugh (1996), Campbell and Viceira (1999), and many others. The certainty equivalent is the sure amount of wealth a risk-averse investor must receive to compensate them for not being able to invest in the potentially decoupling market. Equivalently, this is the maximum amount of money an investor would be willing to pay to invest in the potentially decoupling market. Put another way, not investing in the potentially decoupling market is a portfolio restriction on the investor's opportunity set. Imposing any constraint on the agent's asset allocation problem is costly, and the certainty equivalent is the sure amount of money to compensate the investor for being subject to the restriction of not investing in the potentially decoupling market.

We find the certainty equivalents can be large. For one calibrated model that starts with a myopic 35% target allocation to a market which may decouple, the 1-year certainty equivalent is 0.23 cents per dollar of wealth rising to a very large 5.49 cents for an investor with a 10-year horizon for an investor starting in the Fully Investable regime. With no leverage limits imposed, the certainty equivalent cost falls to 0.24 cents at the 10-year horizon, which is still significant relative to the typical expected return of a multi-asset portfolio (see, for example, He *et al.*, 2022).

The model allows us to compute certainty equivalents as we vary probabilities of decoupling, spending rules, and leverage. Certainty equivalents are a non-linear function of the probability of becoming decoupled. In fact, at the 1-year horizon, certainty equivalents can *increase*—although the effects of the hedging demands are relatively small—as the probability of decoupling increases because of hedging demands, consistent with the optimal holdings of the potentially decoupling asset increasing with horizon. Even with large probabilities of becoming decoupled of 0.2 to 0.3 per year, the willingness-to-pay to invest in the potentially decoupling market can exceed three cents per dollar of wealth for the 10-year horizon. As the consumption rates increase, willingness-to-pay decreases as there is effectively less time for the decoupled market to contribute to portfolio returns, but the willingness-to-pay is still approximately two cents per dollar of wealth for a 10% annual payout rate.

Our framework falls into a large optimal asset allocation literature beginning with Samuelson (1969) and Merton (1969, 1971). A key finding in the literature, which is also exhibited in our setting, is that time-varying expected returns can imply differences between one-period (or myopic) portfolio weights and long-horizon portfolio weights in settings where an investor can optimally rebalance each period (see, for example, Kandel and Stambaugh, 1996; Campbell and Viceira, 1999; Campbell *et al.*, 2003). While the original literature assumed that asset returns follow recurrent Markov data-generating processes, more recent papers using technical continuous-time frameworks like Bo *et al.* (2010) and Muller (2022) have incorporated default risk, but in a setting without time-varying regimes. In contrast to these approaches, we use discrete-time Markov data-generating processes with an absorbing state to model the potential decoupling of an emerging market.

Our model follows Ang and Bekaert (2002) who were the first to examine portfolio choice with persistent Markov regimes, but they do not specify one regime where a market decouples as an absorbing state.[3] The literature following Ang and Bekaert (2002) does not consider the specific setting of our calibrations to MSCI Russia or MSCI China Index returns with the MSCI EM Index, and does not model decoupling as an absorbing Markov chain. Another literature following Brown *et al.* (1995) and Jorion and Goetzmann (1999) documents that securities markets can and do disappear and this affects estimates of risk and return, and Bekaert and Harvey (1995) show that countries undergo periods of integration and segmentation with international markets.[4] These papers do not focus on implications for optimal asset allocation for countries that may potentially decouple.

The rest of this paper is organized as follows. We focus the methodology and calibrations to a hypothetical example of a potentially decoupling market representing an emerging market and EM MSCI Index returns. Section 2 starts with estimating a two-state Markov regime-switching model.

We introduce the probability of decoupling in Section 3 and describe the optimal asset allocation problem in Section 4. We report empirical results in Section 5. In Section 6, we repeat the analysis for another hypothetical emerging market example and EM Index returns. Section 7 concludes.

2 Two-State Markov Model

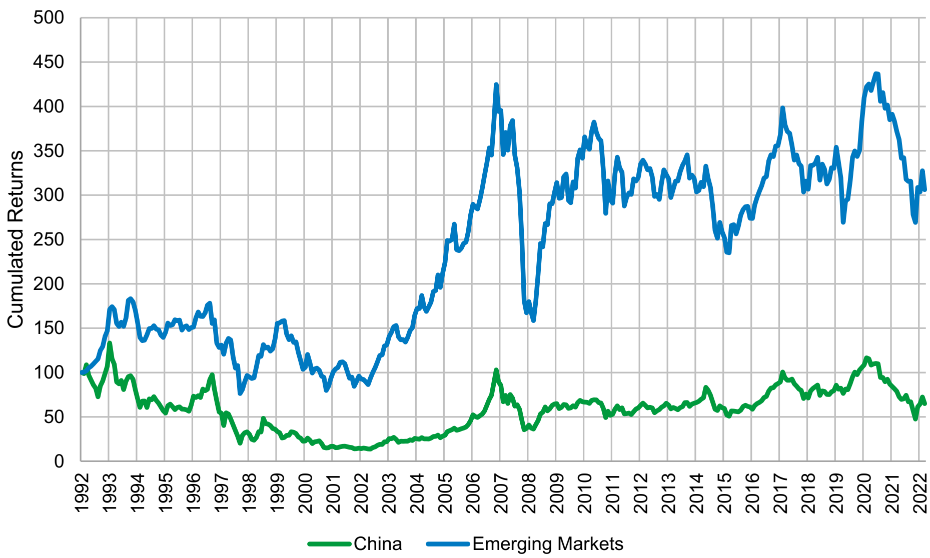
We describe summary statistics of an emerging market country and the MSCI EM Index and estimate a two-state Markov model—which represent the Fully Investable and Potentially Decouple regimes where the probability of becoming decoupled is zero. This sets the stage for Section 3, where we introduce a non-zero probability of decoupling.

2.1 Data

We use the example of China as a market which may potentially decouple.[5] Exhibit 1 graphs cumulated returns of the MSCI China Index returns and the MSCI EM Index from January 1993 to February 2023. Over this period, the mean arithmetic return of China is a low 3.6% per year compared to 6.2% for the EM index. China returns have also been more volatile, at 32.1% per year, relative to EM returns at 21.9%. China's spectacular economic growth taking it from below 10% of US GDP in 1980 to overtaking the US and becoming the largest economy in the world in 2014 measured in purchasing power parity terms according to the World Bank,[6] has not been reflected in large China stock market gains. Allen *et al.* (2022) refer to this as the "disconnect" puzzle, which they attribute to institutional structure and weak corporate governance. Nevertheless, we show below that an investor would optimally allocate to China in certain regimes.[7]

In the 1990s, China constituted a small proportion of the EM Index, but more recently it has reached upward of 40% in September 2020 and is currently around 35%.[8] Thus, China's weight in the EM Index means we may overstate the diversification benefits of investing in China. (The correlation of China returns with EM returns is 0.72 over the sample.) In our model, this is not a

Exhibit 1. Cumulated returns of MSCI China and MSCI Emerging Markets Indexes.



Source: BlackRock, with data from Bloomberg as of March 31, 2023.

concern as we work with a given target allocation to China to infer risk aversion; different starting weights in China will result in different risk aversion levels. On the other hand, we can interpret the investor's opportunity set as two funds, one tracking the MSCI China Index and one tracking the MSCI EM Index and we solve for the optimal weights in each strategy.

2.2 *Markov model*

We estimate the following regime-switching model on continuously compounded returns, $r_t = (r_{1t}\ r_{2t})$, where r_{1t} and r_{2t} are the continuously compounded returns of China and the EM Index, respectively:

$$r_t = N(\mu(s_t), \Sigma(s_t)), \tag{1}$$

where there are different Normal distributions depending on the regime s_t, with the mean, $\mu(s_t)$, and covariance, $\Sigma(s_t)$, dependent on regime s_t.

There are two Markov regimes $s_t = 1, 2$ with the following transition probability matrix:

$$\begin{bmatrix} P & 1-P \\ 1-Q & Q \end{bmatrix}, \tag{2}$$

where $P = Pr(s_{t+1} = 1 \mid s_t = 1)$ and $Q = Pr(s_{t+1} = 2 \mid s_t = 2)$. This is a regime-switching model of Hamilton (1989) which has been used by many researchers to capture non-linearities of equity returns (see the literature review by Ang and Timmermann, 2012).

Exhibit 2 reports the estimates, where the state-dependent means, $\mu(s_t)$, and the state-dependent volatilities, $\sigma(s_t)$, have been annualized by multiplying by 12 and $\sqrt{12}$, respectively. In the first regime, $s_t = 1$, China and EM have approximately the same mean at 8% per year. In the second regime, $s_t = 2$, China exhibits a steeper drawdown, at -19.5% compares to EM at -5.4%.

Exhibit 2. Estimated Markov switching parameters.

	$s_t = 1$	$s_t = 2$
μ_1 China	0.085	−0.195
μ_2 EM	0.087	−0.054
σ_1 China	0.206	0.451
σ_2 EM	0.172	0.294
Correlation	0.822	0.672
	P	Q
Transition probabilities	0.980	0.963

Source: BlackRock, with data from Bloomberg as of March 31, 2023.

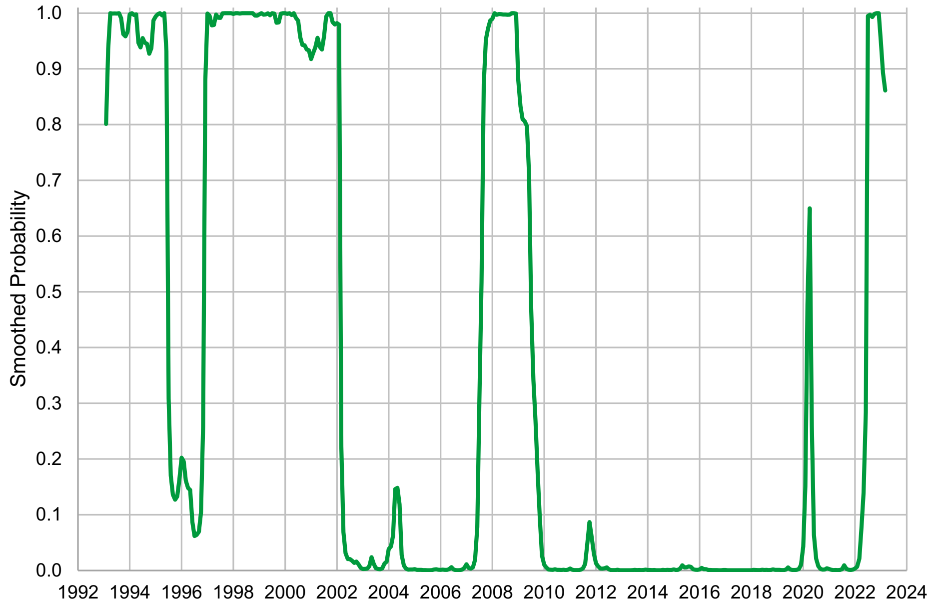
Thus, the first regime corresponds to a "bull" market state for both China and EM returns, whereas the second regime corresponds to "bear" market conditions for both China and EM, with China drawing down four times as much as the EM Index. Both regimes are persistent, with the probability of staying in regime 1 (2) equal to 0.98 (0.96) conditional on the first (second) regime. Important for our calibration, there is a close match of the model-implied arithmetic moments to the data. For example, the implied arithmetic mean returns from the regime-switching model are 3.75% and 6.32% per year for China and EM, respectively, which are close to the arithmetic returns in data at 3.63% and 6.18%, respectively.

Exhibit 3 graphs the smoothed probabilities of the estimation, which is the probability of the bear market $s_t = 2$ conditioning on full-sample information. The second bear market regime occurs during the emerging markets defaults of the late 1990s (see also Exhibit 1), during the Financial Crisis, and more recently ticks upward during the corona virus period of 2020 and at the end of 2022.

3 Markov Chain Model with Decoupling

To introduce the probability of China becoming decoupled, we introduce one more state, $s_t = 3$,

Exhibit 3. Smoothed probabilities of regime $s_{t=2}$.



Source: BlackRock, with data from Bloomberg as of March 31, 2023.

and an additional transition probability, ψ, from regime $s_t = 2$ to regime $s_{t+1} = 3$. The estimation of the two-state Markov chain in the previous section is conditional on China not becoming decoupled (that is, $\psi = 0$). The three-state Markov chain transition probabilities are given by:

$$\begin{bmatrix} P & 1-P & 0 \\ 1-Q-\psi & Q & \psi \\ 0 & 0 & 1 \end{bmatrix}, \tag{3}$$

where $P = Pr(s_{t+1} = 1 \mid s_t = 1)$, $Q = Pr(s_{t+1} = 2 \mid s_t = 2)$, and $\psi = Pr(s_{t+1} = 3 \mid s_t = 2)$. Once the system enters the third Decoupled regime, the system remains in that regime, $Pr(s_{t+1} = 3 \mid s_t = 3) = 1$, so $s_t = 3$ is an absorbing state. In the Decoupled regime, we assume that investors lose all their capital in the first asset.

We take the parameters of the first regime, $s_t = 1$, from the first regime of the two-state model in Exhibit 2. Since in the transition probability matrix in Equation (3), there is no transition to becoming decoupled over the next month, $s_t = 1$ is the "Fully Investable" regime.

In the second "Potentially Decouple" regime, $s_t = 2$, there is a possibility of China becoming decoupled next period with probability ψ. As a baseline case, we set ψ to correspond to an annual probability of 0.05, so $\psi = 1 - \exp(\log(0.05/12)) = 0.0043$ per month. (Below, we examine sensitivity of results to different ψ.) The parameters of the two-state Markov model in Exhibit 2 correspond to $\psi = 0$. When we introduce non-zero ψ, the moments of the regime-switching model change. In the second regime of the two-state Markov model, the gross return for the first asset, China, is $\bar{R}_1(s_t = 2) = \exp\left(\mu_1(s_t = 2) + \frac{1}{2}\sigma_1^2(s_t = 2)\right) = 0.9923$ per month, where $\mu_2(s_t = 2)$ and $\sigma_1(s_t = 2)$ are the per month values given from the regime $s_t = 2$ parameters for China in Exhibit 2. We keep the gross return conditional on $s_t = 2$ the same, so for a given value of ψ, we solve for $\mu_1(s_t = 2)$

such that $\bar{R}_1(s_t = 2)$ is kept constant:

$$\begin{aligned}\bar{R}_1(s_t = 2) &\\ = (1 - Q - \psi)&\\ \times \exp\left(\mu_1(s_t = 1) + \frac{1}{2}\sigma_1^2(s_t = 1)\right)&\\ + Qexp\left(\mu_1(s_t = 2) + \frac{1}{2}\sigma_1^2(s_t = 2)\right)&\\ + \psi \times 0. &\quad (4)\end{aligned}$$

The first term on the right-hand side (RHS) of equation (4) is the probability of transitioning from the Potentially Decouple regime back to the Fully Investable regime multiplied by the gross return of the Fully Investable regime. The second term on the RHS of Equation (4) is the probability of remaining in the Potentially Decouple regime multiplied by the gross return of the Potentially Decouple regime—in this term, we implicitly solve for $\mu_1(s_t = 2)$ to yield $\bar{R}_1(s_t = 2)$ on the left-hand side. The last term is included for pedagogy: the probability of transitioning to the Decoupled regime is ψ, and the gross return of the first asset in this regime is zero. It is worth noting that while we are setting the parameters to hold the mean arithmetic return constant, changing $\mu_1(s_t = 2)$ does change the variance of returns—which is precisely the point of introducing the Decoupled regime, where continuously compounded returns for the first asset can jump to $-\infty$ (or the gross return jumps to zero).

4 Asset Allocation with an Absorbing State

In this section, we set out the investor's asset allocation problem.

The investor receives utility, $\mathcal{U}$, over intermediate consumption from months $t + 1$ to T and rebalances their portfolio at each time s:

$$\mathcal{U}(W_t, s_t, T) = \sum_{s=1}^{T} \beta^s U(c_{t+s} W_{t+s}), \quad (5)$$

where we set the monthly time discount $\beta = (0.98)^{1/12}$ (see Cohen *et al.*, 2020). The terminal condition is $c_{t+T} = 1$. We express consumption as a rate, c_t, of current wealth, W_t. A fraction, $c_t W_t$, of the portfolio is consumed at each time t. We will exogenously set a constant consumption rate $c_t = \bar{c}$.[9]

We assume Constant Relative Risk Aversion (CRRA) utility:

$$U(c_t W_t) = \frac{(c_t W_t)^{1-\gamma}}{1-\gamma}, \quad (6)$$

where γ is the investor's coefficient of risk aversion.

The investor's wealth dynamics are given by:

$$W_{t+1} = (1 - c_t) W_t (\omega_t R_{1t+1} + (1 - \omega_t) R_{2t+1}), \quad (7)$$

where ω_t is the fraction of the investor's portfolio held in the first asset chosen at the beginning of the period t. The gross return of the i-th asset is $R_{it+1} = R_{it+1}(s_{t+1})$ over time t to $t + 1$ and depends on the regime prevailing at time $t + 1$, s_{t+1}. The probabilities of the regime at time $t + 1$, s_{t+1}, depend on the current regime at time t, s_t. If the system is in the Potentially Decouple regime $s_t = 2$ at time t, it may transition to being decoupled at time $t + 1$. If the first asset has decoupled at time t (and thus is in regime $s_t = 3$), it remains decoupled at time $t + 1$, and the regime $s_{t+1} = 3$ with probability one. In the Decoupled regime, the gross return of the first asset is equal to zero.

While the academic literature usually solves the portfolio weights, $\{\alpha_t\}$, and consumption stream, $\{c_t\}$, simultaneously (see, for example, Garlappi and Skoudakis, 2010), we take the consumption rate as given and in our baseline calibrations set $\bar{c} = c_t = 0.05/12$ at the monthly frequency. This is consistent with industry practice where payout rates tend to be fixed or liabilities are exogenous. For example, endowments tend to pay out at least 5% per year to preserve their tax-exempt status (see Brown and Tiu, 2013), financial advisors recommend a fixed spending rule, often 4%, for a retired person (see Bengen, 1994), and income funds often target fixed payout rates like 7% or more which cater to individuals' demands for stable cashflows (see Harris *et al.*, 2015).

There are three regimes in the Markov chain: Fully Investable, Potentially Decouple, and Decoupled, so at each horizon, T, there are three optimal portfolio weights corresponding to each regime, that is, $\omega_t = \omega_t(T, s_t)$. In the Decoupled regime, $s_t = 3$, it is only possible to allocate to the EM Index.

To calibrate the investor's risk aversion, we assume a target China holding of 35% for the first Fully Investable regime, $s_t = 1$, for a myopic horizon of $T = 1$ month. This corresponds to a risk aversion of $\gamma = 0.662$. Below, we also conduct exercises varying the investor's risk aversion.[10]

Using the asset allocation framework, we compute the certainty equivalent required to compensate an investor for not being able to invest in China, where the restricted portfolio consists only of the EM asset. The certainty equivalent depends on the prevailing regime, s_t, and the investor's horizon, T. In our calculations, we also compute solutions limiting the amount of leverage.

5 Empirical Results

In this section, we report the empirical results. Section 5.1 reports the optimal allocations to the decoupling asset with and without leverage constraints. Section 5.2 reports certainty equivalents of not investing in the decoupling market. In Sections 5.3 to 5.6, we consider various comparative statics exercises changing the probability of decoupling, the consumption payout ratio, maximum leverage, and the decoupling asset's expected returns, respectively.

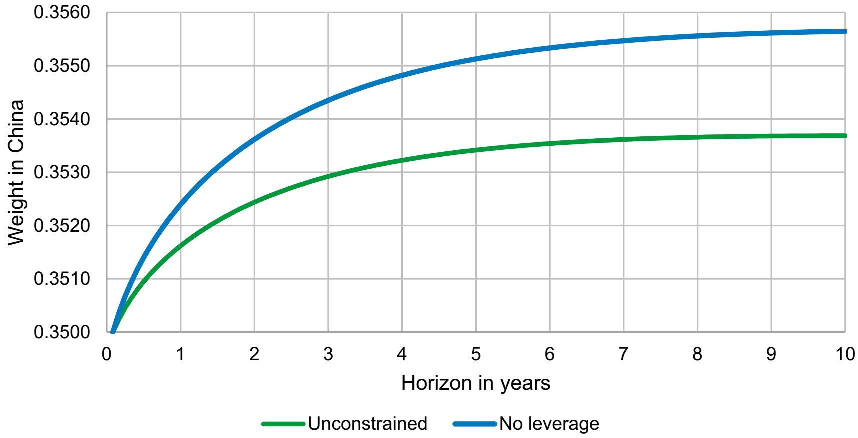
5.1 Optimal allocations

Exhibit 4 reports the optimal holdings for China in the Fully Investable regime and the Potentially Decouple regime in Panels A and B, respectively. These are produced under our baseline parameters: risk aversion of $\gamma = 0.662$, 5% annual payout ratio, and a 1-year probability of becoming decoupled of 0.05. We consider two cases: an unconstrained case that allows shorting and a no-leverage case.

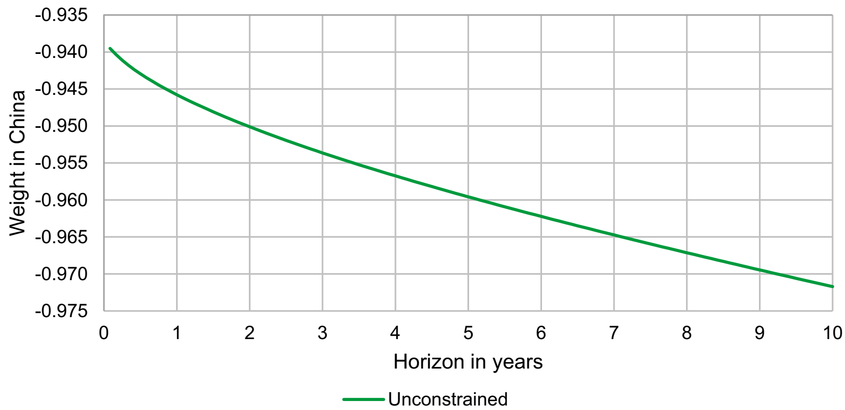
In Exhibit 4 for both the unconstrained and no-leverage cases, the starting myopic portfolio holding of 35.0% in China is by construction (as this is the target weight to calibrate the risk aversion parameter). As the horizon increases, the investor seeks to hold a larger amount of China—indicating that there are intertemporal hedging demands. At the 5-year horizon, the unconstrained investor optimally holds 35.4% in China and at the 10-year horizon, the unconstrained investor holds 35.6%. Although the hedging demands are small—increasing from 35.0% at the myopic weight to 35.6% at the 10-year horizon, it may seem unintuitive that the optimal weights increase as the longer the horizon, the higher the cumulative probability of the asset to become decoupled! These results are consistent with predictability arising from the regime persistence (see Exhibit 2 for the two-state Markov model, for example). The longer the horizon, the more consumption is foregone from not investing in the first asset. Furthermore, the longer the horizon, the more the investor can take advantage of the persistent regimes.

Exhibit 4. Optimal holdings of China.

Panel A: Fully Investable regime $s_t = 1$



Panel B: Potentially Decouple Regime $s_t = 2$



The figures shown relate to past performance and are calculated under the model Equations (1)–(7) with the solution method in the Appendix using the parameters in Exhibit 2, $\gamma = 0662$, $\bar{c} = 005/12$, and an annual 1-year decoupling probability of 0.05.

Source: BlackRock, with data from Bloomberg as of March 31, 2023.

In Exhibit 4, Panel A, the effect is smaller for the no-leverage case as horizon increases because the optimizer wishes to short China in the second Potentially Decouple regime, shown in Panel B. In this regime, $s_t = 2$, the investor takes a short position because China may decouple; as a reminder, we have calibrated $\mu_1(s_t = 2)$ so that the arithmetic return of the first asset in the second regime remains the same as the two-state Markov model without decoupling risk, so this short position is caused by the investor anticipating a downward jump to zero on the probability ψ of China becoming decoupled. By construction in the no-leverage case, the weight in China is zero (not graphed), and in the third regime when China becomes decoupled, the investor holds $\omega = 0$ and 100% EM.

5.2 *Certainty equivalent costs*

Exhibit 5 reports the certainty equivalent costs of not investing in China in the Fully Investable

Exhibit 5. Certainty equivalent costs of not investing in China.

	Unconstrained		No-leverage	
	Fully Investable regime	Potentially Decouple regime	Fully Investable regime	Potentially Decouple regime
1 Month	0.0046	0.2893	0.0046	0.0000
2 Months	0.0136	0.5412	0.0086	0.0001
6 Months	0.0790	1.3420	0.0220	0.0015
1 Year	0.2311	2.2550	0.0383	0.0053
3 Years	1.1564	4.8247	0.0911	0.0296
5 Years	2.3142	6.7547	0.1369	0.0604
10 Years	5.4910	10.7410	0.2383	0.1422

The figures shown relate to past performance and are calculated under the model Equations (1)–(7) with the solution method in the Appendix using the parameters in Exhibit 2, $\gamma = 0662$, $\bar{c} = 005/12$, and an annual 1-year decoupling probability of 0.05.

Source: BlackRock, with data from Bloomberg as of March 31, 2023.

($s_t = 1$) and Potentially Decouple ($s_t = 2$) regimes. (Since China has decoupled in the $s_t = 3$ regime, the certainty equivalent cost is zero.) Exhibit 5 shows that the certainty equivalent costs are very high in the case of allowing leverage—0.23 cents per initial dollar of initial wealth for a 1-year horizon increasing to an extremely large 5.49 cents at the 10-year horizon. In the Potentially Decouple regime, the costs are even higher—at 2.26 and 10.74 cents per dollar of wealth for the 1- and 10-year horizons, respectively. This is because the investor takes short positions in China in preparation for a possible decoupling event in the Potentially Decouple regime.

The last two columns of Exhibit 5 list certainty equivalents for the no-leverage case. Not surprisingly, the certainty equivalents are now much smaller than the unconstrained case. In the no-leverage case, the investor does not hold any China exposure in the Potentially Decouple regime (see Section 5.1 and Exhibit 4), but Exhibit 5 reports that the certainty equivalent costs are non-zero in that regime. This is because there is a probability $(1 - Q - \psi)$ of transitioning back to the Fully Investable regime where it is optimal to hold China, and this potential outcome is anticipated and taken into account in the certainty equivalent calculation. The 1-year and 10-year certainty equivalent costs in the Fully Investable regime are 3.8 bp and 24 bp, respectively.

In the next sections 5.3 to 5.6, we vary the baseline parameters to investigate how the certainty equivalent costs change as we change other parameters.

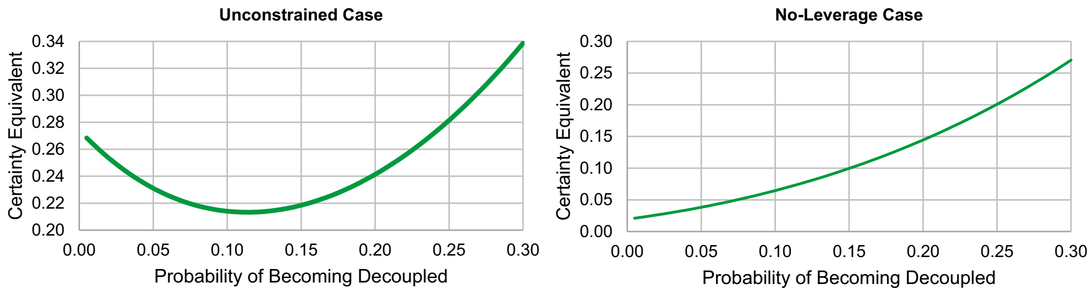
5.3 *Changing the probability of decoupling*

Exhibit 6 plots the certainty equivalents in cents per initial dollar of wealth as a function of horizon, T, for the unconstrained leverage case as we vary the probability of becoming decoupled. We plot the certainty equivalents of not investing in China in the Fully Investable regime. The units on the x-axis are the probability of decoupling over the next year (which are then converted to monthly ψ units in the model). As a reminder, the baseline probability of decoupling used for Exhibits 4 and 5 is 0.05 per year. The other parameters are held at baseline.

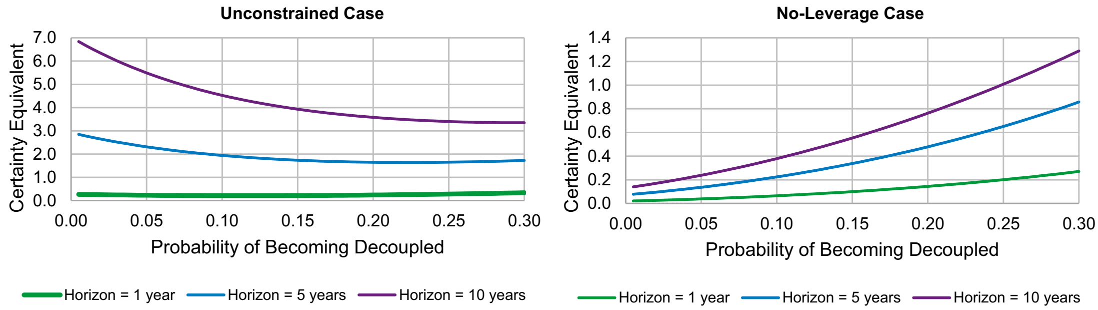
Interestingly, in Panel A the certainty equivalent costs for the 1-year horizon are a non-linear

Exhibit 6. Certainty equivalent costs varying probabilities of decoupling in the Fully Investable regime holding constant the mean of the potentially decouple regime.

Panel A: 1-Year horizon



Panel B: 1-Year, five-year, and 10-year horizons



The figures shown relate to past performance and are calculated under the model Equations (1)–(7) with the solution method in the Appendix using the parameters in Exhibit 2, $\gamma = 0662$, and $\bar{c} = 005/12$. The probability of becoming decoupled is over the next year. The horizons refer to the holding period of the investor.

Source: BlackRock, with data from Bloomberg as of March 31, 2023.

function of horizon for the unconstrained leverage case. There are two offsetting effects: at low probabilities of decoupling, the investor seeks to hold the additional source of return in China with little probability of loss, so the certainty equivalent is relatively high. At high probabilities of decoupling, the investor can take advantage of becoming decoupled by taking short positions in the Potentially Decouple regime, again increasing the certainty equivalent. In the no-leverage case, there is no benefit for the short position and so the certainty equivalents are lower and monotonically increasing.

Why does the certainty equivalent generally increase in both the unconstrained and constrained leverage cases at the 1-year horizon with an *increasing* probability of decoupling? This is caused by two reasons. First, the Fully Investable regime is persistent and the 1-year horizon is relatively short. We require two Markov chain transitions—from the Fully Investable to the Potentially Decouple regimes, and then the Potentially Decouple to the Decoupled regimes—and the investor is more likely not to experience decoupling starting from the Fully Investable regime. Second, we have held the gross return of the Potentially Decouple regime constant (we solve for $\mu_1(s_t = 2)$ as detailed in Section 4 and Equation (3)), and at relatively low levels of risk aversion, the effect of the higher moments of the jump to zero in decoupling is second to the mean effect for CRRA utility (see Pratt, 1964).

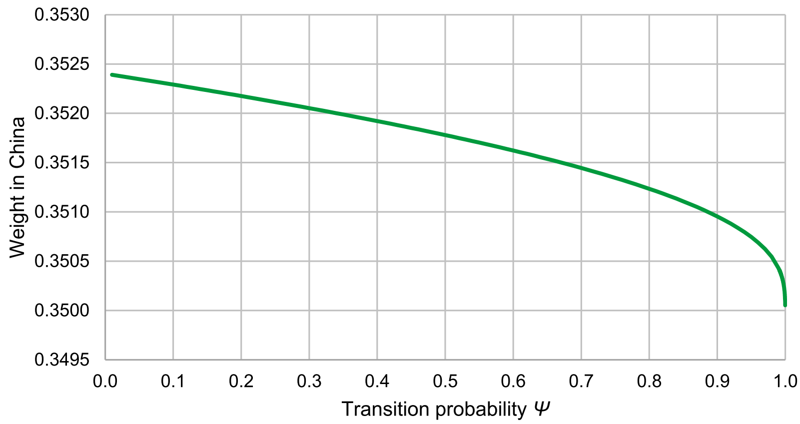
When the horizon is long enough in Exhibit 6, Panel B, the certainty equivalents do decrease for the unconstrained case. In the case of no leverage, however, the certainty equivalents are still an increasing function of the probability of decoupling. In this case, the optimal holdings of China in the second and third regime are already zero. These constrained positions, combined with the two necessary regime changes to become decoupled (Fully Investable to Potentially Decouple to Decoupled) mean that the decoupling asset is a relatively attractive asset in the Fully Investable regime.

5.4 *Changing the probability of decoupling without adjusting the first moment*

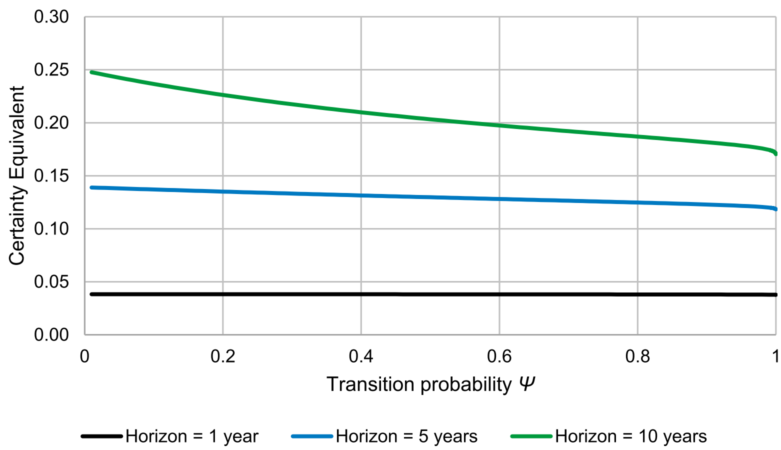
The previous exercise ensured comparability of the different probabilities of decoupling by making a transformation to the mean of the

Exhibit 7. Changing the transition probability of becoming Decoupled, ψ, no leverage case.

Panel A: Weight of China for a 1-year horizon in the Fully Investable regime $s_t = 1$



Panel B: Certainty equivalent costs in the Fully Investable regime $s_t = 1$



The figures shown relate to past performance and are calculated under the model Equations (1)–(8) with the solution method in the Appendix using the parameters in Exhibit 2, $\gamma = 0662$, $\bar{c} = 005/12$, and setting $\mu_1(s_t = 1)$ to correspond to an annual 1-year decoupling probability of 0.05 following Equation (4). The probability of transitioning to the Decoupled regime, ψ, is varied following Equation (8).

Source: BlackRock, with data from Bloomberg as of March 31, 2023.

Potentially Decouple regime (see Equation (4)) so that the gross return in that regime was held constant. This highlighted the effect of the probability of decoupling on risk. We now investigate changing the transition probability from the Potentially Decouple regime to the Decoupled regime, ψ, so that it affects both risk and return.

In our analysis, we take the baseline value of $\mu_1(s_t = 2)$ that was used in Exhibits 4 and 5 corresponding to a 1-year probability of decoupling of 0.05 (see also the parameter values in Exhibit 2). We set $\gamma = 0.662$, and use a 5% annual payout ratio. We specify the transition probability matrix to be:

$$\begin{bmatrix} P & 1-P & 0 \\ (1-\psi)(1-Q) & (1-\psi)Q & \psi \\ 0 & 0 & 1 \end{bmatrix}, \quad (8)$$

so that with probability $(1 - \psi)$ the system remains investable in the Fully Investable or Potentially Decouple regimes. From the Potentially Decouple regime, there is probability ψ that the system enters the absorbing state of Decoupled.

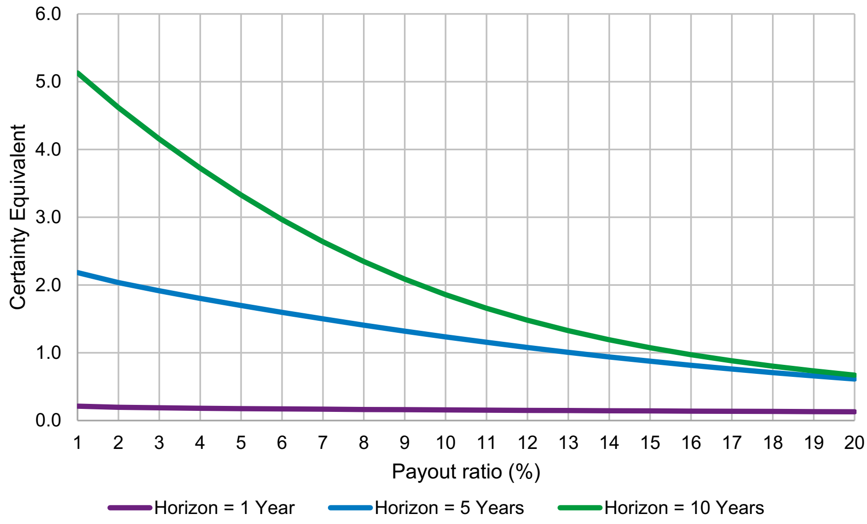
Exhibit 7 reports the results for the no leverage case. Panel A shows the 1-year asset allocation weight in China in the Fully Investable regime—the weights are remarkably constant around 0.35 for all but the values of ψ very close to one (note the weight in China asymptotes to zero as $\psi \to 1$). The reason is that we start in the Fully Investable regime, which has a probability of remaining in that regime of 0.982 (see Exhibit 2) and the optimal policy is to invest in China in that regime. As $\psi \to 1$, the Potentially Decouple and the Decoupled regime effectively merge and the data-generating process can then be interpreted as returns being log-normally distributed but subject to occasional jumps where the investor can lose all capital in the potentially decoupling market.[11] As long as $0 < \psi < 1$, however, the presence of the Potentially Decouple regime induces time-varying decoupling probabilities.

Panel B of Exhibit 7 reports certainty equivalent costs of not investing in China in the Fully Investable regime as a function of ψ.[12] Certainty equivalent costs do approach zero when ψ becomes very close to one, but the effect is not visible in the graph. For the 10-year horizon, the certainty equivalent decreases from approximately 0.25 for ψ close to zero to 0.16 for ψ close to one. For shorter horizons, there is little effect of ψ on the certainty equivalents (except for extreme values of ψ approaching zero or one). The intuition behind this is the same as above—as we are starting in the Fully Investable regime, the large persistence of this regime means that there are non-negligible optimal holdings of China.

5.5 *Changing the payout ratio*

In Exhibit 8, we vary the consumption rate, $c_t = \bar{c}$, or payout ratio for the unconstrained case holding other parameters fixed at baseline. The certainty equivalents become smaller as the payout ratio increases: for example, at the 10-year horizon, the certainty equivalent of not investing in the decoupling asset is over 5 cents per dollar of initial wealth for a 1% consumption rate decreasing to 0.67 cents at a 20% consumption rate. As Wachter (2002) notes, introducing consumption into a CRRA portfolio choice problem effectively shortens the effective horizon, or decreases duration. Lower payout ratios imply higher duration, and although the cumulative probability of China becoming decoupled increases with longer horizons, there is a greater time for the agent to accrue consumption—translating into higher certainty equivalents for lower payout ratios.

Exhibit 8. Certainty equivalent costs varying payout ratio.



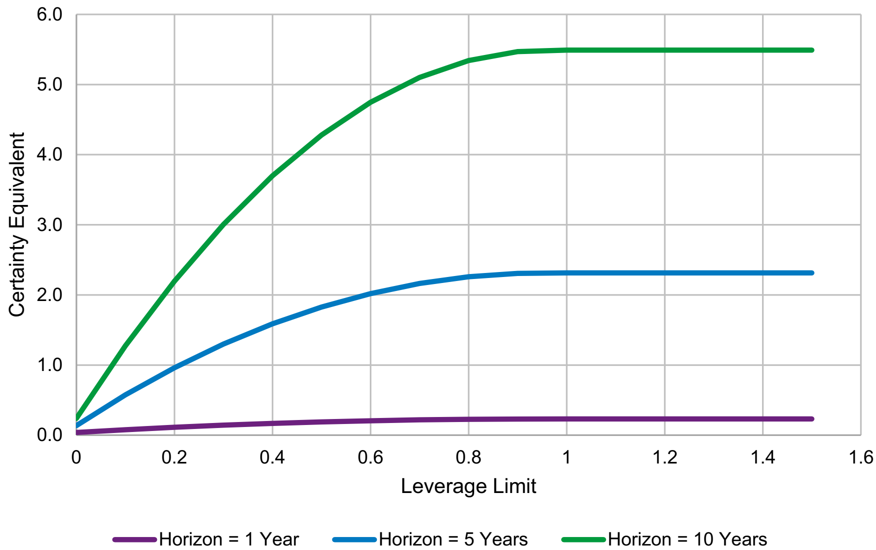
The figures shown relate to past performance and are calculated under the model Equations (1)–(7) with the solution method in the Appendix using the parameters in Exhibit 2, $\gamma = 0.662$ and an annual 1-year decoupling probability of 0.05.

Source: BlackRock, with data from Bloomberg as of March 31, 2023.

5.6 *Changing the leverage limit*

Our next comparative statics exercise is to change the leverage limit in Exhibit 9. The maximum short position for the baseline parameters is approximately -0.9, which is taken in the Potentially Decouple regime, $s_t = 2$. Thus, the certainty equivalents asymptote at this leverage level as there are no more benefits to increasing

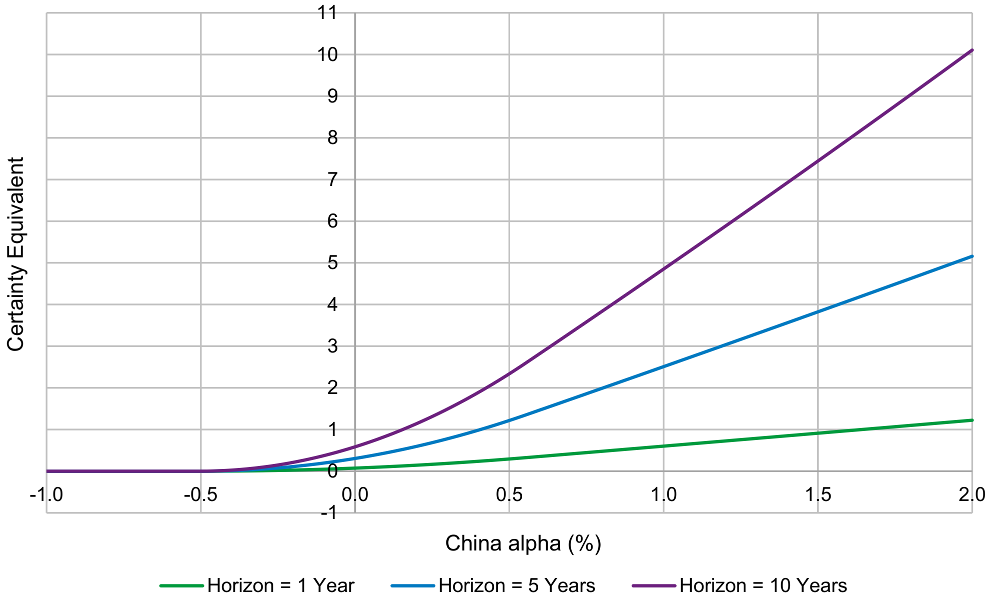
Exhibit 9. Certainty equivalent costs varying the leverage limit.



The figures shown relate to past performance and are calculated under the model Equations (1)–(7) with the solution method in the Appendix using the parameters in Exhibit 2, $\gamma = 0662$, $\bar{c} = 005/12$, and an annual 1-year decoupling probability of 0.05.

Source: BlackRock, with data from Bloomberg as of March 31, 2023.

Exhibit 10. Certainty Equivalent costs varying the alpha of the decoupling asset, no leverage case.



The figures shown relate to past performance and are calculated under the model Equations (1)–(7) and varying the expected return of the potentially decoupling asset following Equation (9), with the solution method in the Appendix using the parameters in Exhibit 2, $\gamma = 0662$, $\bar{c} = 005/12$, and an annual 1-year decoupling probability of 0.05.
Source: BlackRock, with data from Bloomberg as of December 7, 2023.

the leverage limit after this value. Before this, the leverage limit is binding and the utility, and thus certainty equivalents, increase up until this value.

5.7 *Changing the decoupling asset's expected return*

Our last exercise is to investigate the sensitivity of the certainty equivalent and the decoupling asset's expected returns. We specify that in the Fully Investable regime, the expected return of China to be equal to the expected return of EM plus an alpha term:

$$\mu_1(s_t = 1) = \alpha + \mu_2(s_t = 1). \qquad (9)$$

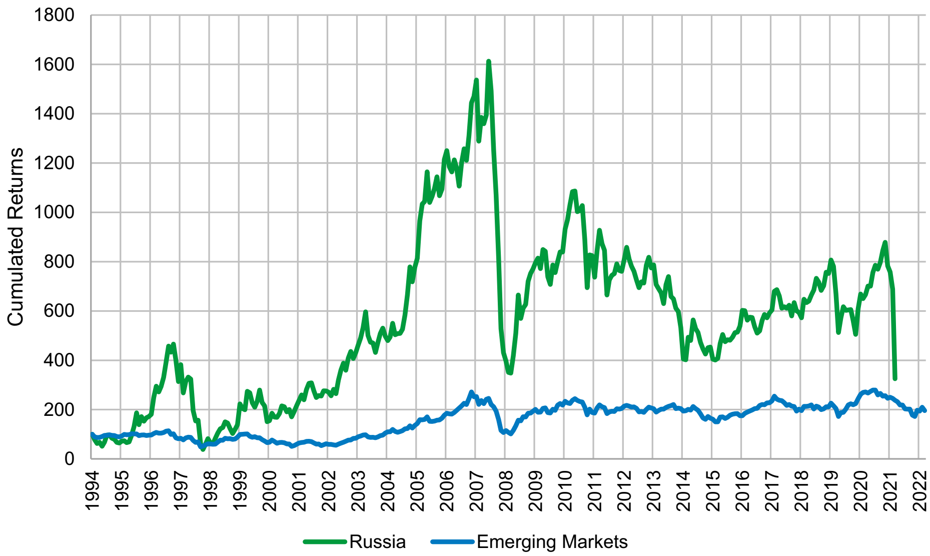
Note that in the baseline calibration (see Exhibit 2), the expected return of China and EM is approximately the same—with $\mu_1(s_t = 1) = 0.085$ and $\mu_2 = 0.087$ per year for China and EM, respectively, and thus in our full data sample, α is approximately equal to zero.

Exhibit 10 reports the certainty equivalents as a function of α for the no leverage case, where α is annualized. The no leverage limit binds for α approximately below -0.4% per year, so the certainty equivalents are zero. As alpha becomes more positive, the optimal weight in China increases and the certainty equivalents increase. At an annualized alpha of 1%, the 1-year certainty equivalent is 0.60 cents per initial dollar of wealth, and the 5-year and 10-year horizon certainty equivalents are 2.5 and 4.9 cents per initial dollar of wealth, respectively. At 2% alpha, the 10-year horizon certainty equivalent is above 10 cents.[13]

6 Calibration with Negative Hedging Demands

In this section, we examine optimal asset allocation to MSCI Russia and the MSCI EM Indexes.

Exhibit 11. Cumulated returns of MSCI Russia and MSCI Emerging Markets Indexes.



Source: BlackRock, with data from Bloomberg as of March 31, 2023.

Russia decoupled in March 2022, so that these hypothetical exercises can be interpreted as applying prior to this date. Exhibit 11 plots cumulated returns of Russia and EM from January 1995 to February 2022. Prior to Russia decoupling, Russia experienced steep negative returns at the beginning of 2022—suggestive of the Potentially Decouple regime, and then Russia moved to the Decoupled regime in March 2022. The returns since Russia liberalized to when it decoupled were high—at 16.5% per year compared to 5.7% per year for the EM Index. Returns of Russia were also approximately twice as volatile, at 48.4%, as the EM Index, at 21.9%.

We follow the same procedure as the previous section, where we model China as an asset that may potentially decouple. We fit a two-state Markov chain model to Russia and EM, and then introduce a third absorbing state where Russia becomes decoupled. The parameters of the regime-switching model have the same qualitative characteristics as the model estimated on China and EM in Section 2—in the second regime Potentially Decouple, the returns of Russia are worse than EM and are also more volatile. This leads to an investor wanting to take modest short positions in Russia in the Potentially Decouple regime.

We calibrate the risk aversion to a target allocation of Russia of 5% in the Fully Investable regime, which corresponds to $\gamma = 10.0$—this is much larger than the calibrated risk aversion in the China-EM system ($\gamma = 0.67$) because the historical returns of Russia are much higher than China (up until Russia became decoupled). We hold the other parameters the same as the China case.

In Exhibit 12, we report portfolio weights and certainty equivalent costs for not investing in Russia. The portfolio weight starts, by construction, at 5.0% and the optimal portfolio weights decrease; this is unlike the China case because the probability of decoupling is high. The portfolio weights asymptote at -2.8%. The short positions are higher in the Potentially Decouple regime, as expected, as the optimizer seeks to take advantage of the possibility that Russia can transition to being decoupled. The certainty

Exhibit 12. Certainty equivalent costs of not investing in Russia.

	Portfolio weights		Certainty equivalent costs	
	Fully Investable regime	Potentially Decouple regime	Fully Investable regime	Potentially Decouple regime
1 Month	0.0500	−0.0772	0.0000	0.0000
2 Months	0.0500	−0.0772	0.0028	0.1020
6 Months	0.0260	−0.0697	0.0441	0.4960
1 Year	−0.0102	−0.0606	0.3847	0.9963
3 Years	−0.0276	−0.0504	2.2383	2.4117
5 Years	−0.0276	−0.0498	3.5256	3.6962
10 Years	−0.0276	−0.0498	6.7914	6.9673

The figures shown relate to past performance and are calculated under the model Equations (1)–(7) with the solution method in the Appendix with model parameters calibrated to the historical returns in Exhibit 9, $\gamma = 100$, $\bar{c} = 005/12$, and an annual 1-year decoupling probability of 0.05.

Source: BlackRock, with data from Bloomberg as of March 31, 2023.

equivalents are negligible at the 1-month horizon and rise to 6.8 cents per dollar of initial wealth at the 10-year horizon. The certainty equivalents are approximately the same across the Fully Investable regime and the Potentially Decouple regime because the probability of decoupling is relatively high.

7 Conclusion

We model optimal allocations to markets that may become decoupled. The investor receives utility over intermediate consumption, which is funded by liquidating the investment portfolio. While the market is still investable, holdings of the market that may potentially decouple represent a source of returns and diversification to partially fund consumption. We model time-varying investment returns with a Markov switching model with three Markov regimes—a Fully Investable regime where there is no possibility of the market becoming decoupled, a Potentially Decouple regime where the market can transition to the third regime, which is an absorbing state, the Decoupled regime where the market experiences a −100% return. The regimes are persistent, which induces hedging demands, or where long-horizon portfolio weights are different from short-horizon portfolio weights.

Using the model, we compute certainty equivalents—the sure amount of money required to compensate the investor for not being able to invest in the market that may become decoupled—and find these are large and increase with horizon when calibrating the model with an emerging market and the MSCI Emerging Market Index. In particular, we find that hedging demands, although small, are positive: optimal holdings of the potentially decoupling market can increase with horizon in the Fully Investable regime and when the probability of becoming decoupled increases.

Because there are two non-decoupled regimes, the probability of decoupling already varies through time. However, it is likely that the probability of decoupling may be a function of macro variables. An interesting and serious extension of the model is to introduce time-varying transition probabilities as a function of exogenous predictors along the lines of Diebold *et al.* (1999) or Bazzi *et al.* (2016). In this case, there would be additional state variables in the asset allocation problem making the dynamic programming solution much more involved. It is a straightforward extension to allowing the potentially decoupling market to re-integrate into world capital markets along the lines of Bekaert and Harvey (1995). In this case, we would assume some potential loss in the Decoupled regime, which could be stochastic, and the Decoupled regime would no longer be an absorbing state. Another application is in the field of sustainability: under some scenarios in the transition to a low carbon economy, certain assets may become "stranded" and lose their value—for example, those assets associated with fossil fuels (see a review by Van der Ploeg and Rezai, 2020, for example). The date at which these assets may become stranded is unknown, so the data-generating process for an asset that may become stranded may be suited to modeling by an absorbing Markov chain as in this paper.

APPENDIX

We solve the dynamic asset allocation problem using dynamic programming. The Bellman equation for the system in Equations (5)–(7) can be written as

$$\frac{\vartheta(T, s_t)}{1-\gamma} = \max_{\omega_t}\left[\frac{c_t^{1-\gamma}}{1-\gamma} + \beta E_t \times\left[\frac{((1-c_t)W_t(\omega_t R_1 + (1-\omega_t)R_2))^{1-\gamma}}{1-\gamma} \times \vartheta(T-1, s_{t+1})\right]\right], \quad \text{(A.1)}$$

where $\vartheta(T, s_t)$ is a function of horizon T and state s_t and $R_i = R_i(s_{t+1})$ is the return for asset i which is a function of the regime prevailing at time $t+1$, s_{t+1}. This is a simpler problem than the standard CRRA intermediate consumption problem because we take the consumption rate, c_t, as given. However, as Brandt (2010) notes, with CRRA utility the value function is homothetic in wealth and the consumption choice is solved independently of the portfolio weight, ω. That is, even with endogenous consumption, the portfolio choice weight at time t, ω_t, is solved ignoring the consumption rate, and then the optimal consumption rate is solved taken as given the optimal portfolio weight.

The first-order conditions (FOCs) are given by:

$$E_t[(\omega_t R_1 + (1-\omega_t)R_2)^{-\gamma} \times \vartheta(T, s_{t+1})(R_1 - R_2)] = 0. \quad \text{(A.2)}$$

In Equation (A.2), $(R_1 - R_2)$ is the return of the first asset relative to the second asset. There are three regimes at time $t+1$, $s_{t+1} = 1, 2$, or 3, whose probabilities depend on the current regime at time t, s_t, and hence there are three sets of FOCs. Likewise the value function, $\vartheta(\tau, s_t)$ takes on three values for each $\tau = T \ldots 1$, one for each state, s_t. The boundary conditions are $\vartheta(T, s_t) = 1$ for all s_t.

The FOC conditional on $s_t = 1$ is given by:

$$\begin{aligned}(1-P)E_t[W_{t+1}^{-\gamma}(s_{t+1})\vartheta(s_{t+1}, T-1) &\times (R_1(s_{t+1}) - R_2(s_{t+1})) \mid s_{t+1} = 1] \\ + PE_t[W_{t+1}^{-\gamma}(s_{t+1})\vartheta(s_{t+1}, T-1) &\times (R_1(s_{t+1}) - R_2(s_{t+1})) \mid s_{t+1} = 2] = 0,\end{aligned} \quad \text{(A.3)}$$

where $W_{t+1}^{-\gamma}(s_{t+1}) = (\omega R_1(s_{t+1}) + (1-\omega)R_2(s_{t+1}))^{-\gamma}$ and $P = \Pr(s_{t+1} = 1 \mid s_t = 1)$ in the probability transition matrix in Equation (3). We solve the non-linear root for Equation (A.2) to find $\omega_t(s_t = 1)$.

The FOC conditional on $s_t = 2$ is given by:

$$(1-Q-\psi)E_t[W_{t+1}^{-\gamma}(s_{t+1})\vartheta(s_{t+1}, T-1) \times (R_1(s_{t+1}) - R_2(s_{t+1})) \mid s_{t+1} = 1]$$

$$+ QE_t[W_{t+1}^{-\gamma}(s_{t+1})\vartheta(s_{t+1}, T-1)$$
$$\times (R_1(s_{t+1}) - R_2(s_{t+1})) \mid s_{t+1} = 2]$$
$$+ \psi E_t[W_{t+1}^{-\gamma}(s_{t+1})\vartheta(s_{t+1}, T-1)$$
$$\times (-R_2(s_{t+1})) \mid s_{t+1} = 3] = 0, \quad \text{(A.4)}$$

where $R_1(s_{t+1} = 3) = 0$ is the decoupled first asset which has -100% return in regime $s_{t+1} = 3$ and $W_{t+1}^{-\gamma}(s_{t+1} = 3) = (R_2(s_{t+1} = 3))^{-\gamma}$ as only the second asset exists in regime $s_{t+1} = 3$. The probability $Q = Pr(s_{t+1} = 2 \mid s_t = 2)$ and $\psi = \Pr(s_{t+1} = 3 \mid s_t = 2)$. Solving Equation (A.3) yields $\omega_t(s_t = 2)$

The only solution for the third regime $s_t = 3$ is for $\omega_t(s_t = 3) = 0$. Stated another way, only the second asset is held when the first asset has become decoupled.

We evaluate the integrals in the FOCs numerically by quadrature, which is very accurate for as few as four or five quadrature points (see Kochenderfer and Wheeler, 2019).

We compute the certainty equivalent after solving for the value function and optimal portfolio weights with and without holding the first asset. The latter corresponds only to the expected utility for holding only EM. We denote the value function with optimal first asset holdings as $\vartheta^*(s_t, T)$ and the value function without the first asset as $\vartheta^\dagger(s_t, T)$. The certain amount of wealth, $\tilde{w}$, required to compensate an investor for not being able to invest in the first asset is given by

$$\tilde{w}(s_t, T) = \left(\frac{\vartheta^*(s_t, T)}{\vartheta^\dagger(s_t, T)}\right)^{1/(1-\gamma)}. \quad \text{(A.5)}$$

Note that the certainty equivalent depends on the current regime, s_t, and the horizon, T. We express the compensation required in cents per dollar of initial wealth, $100 \times (\tilde{w} - 1)$.

Endnote

1 See https://www.bloomberg.com/news/articles/2022-03-02/msci-pulls-russian-equities-out-of-its-key-emerging-market-index.

2 Austria and Russia constituted 5% and 6%, respectively, of total equity market capitalization in 1900, as reported by Dimson *et al.* (2023).

3 See also Ang and Bekaert (2004), Guidolin and Timmermann (2008), Tu (2010), Das *et al.* (2022), and Lewin and Campani (2022). Related studies are Liu *et al.* (2003) and Das and Uppal (2004) who examine the implications of jump processes for optimal asset allocation, except our jump downward is irreversible and set at -100%.

4 We work with a conservative assumption that the decoupling regime is an absorbing state, but allowing for time-varying integration can be modeled with a small probability of exiting the Decoupled regime. Dimson *et al.* (2021) forcefully make the point that "Russia and China are the two best-known cases of markets that failed to survive and in which investors lost everything."

5 Discussion about China potentially decoupling has been present since 2021. See https://www.bloomberg.com/news/articles/2021-07-29/goldman-clients-are-asking-if-china-s-stocks-are-uninvestable.

6 See https://data.worldbank.org/indicator/NY.GDP.MKTP.KD.ZG?locations=CN.

7 We obtain similar results using the MSCI China A Onshore Net Return Index. The MSCI China index and the MSCI China A Onshore Net Return index have similar means and standard deviations over their common sample periods. The parameters of the regime-switching model (see Section 2.2) are similar when using the MSCI China A Onshore Net Return index.

8 See https://www.bloomberg.com/news/articles/2023-01-06/china-set-to-reclaim-third-of-global-em-stock-gauge-on-xi-pivot#xj4y7vzkg.

9 See the Appendix for details on the solution of the model in Equations (5)–(7) and the definition of the certainty equivalent in the context of the model.

10 Unconstrained mean–variance or CRRA utility using the data first and second moments (see Exhibit 1) produces short positions in China for all positive levels of risk aversion because the returns of China are lower and more volatile than the EM Index with correlations that are too high for any mean–variance diversification benefit.

[11] Liu *et al.* (2003), Wu (2003), Das and Uppal (2004), and Ait-Sahalia *et al.* (2009), among others, study asset allocation with IID jump processes.

[12] Naturally, the certainty equivalent costs of the case allowing for leverage are orders of magnitude larger as the investor optimally wishes to short China in the Potentially Decoupling regime, and the short positions become larger in absolute value as ψ becomes larger.

[13] These certainty equivalents are primarily due to the curvature of the utility function itself with low risk aversion. The portfolio weight under the no-leverage constraint lies between 0 and 1 between alphas of -0.50% and 0.56%. There is large sensitivity of the optimal holding to China because the risk aversion in the baseline calibration is low, at $\gamma = 0.662$.

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References

Ait-Sahalia, Y., Cacho-Diaz, J., and Hurd, T. R. (2009). "Portfolio Choice with Jumps: A Closed-Form Solution," *Annals of Applied Probability* **19**(2), 556–584.

Allen, F., Qian, J., Shan, C., and Zhu, J. (2022). "Dissecting the Long-Term Performance of the Chinese Stock Market," SSRN Working Paper.

Ang, A. and Bekaert, G. (2002). "International Asset Allocation with Regime Shifts," *Review of Financial Studies* **15**(4), 1137–1187.

Ang, A. and Bekaert, G. (2004). "How Do Regimes Affect Asset Allocation," *Financial Analysts Journal* **60**(2), 86–99.

Ang, A. and Timmermann, A. (2012). "Regime Changes and Financial Markets," *Annual Review of Financial Economics* **4**, 313–337.

Bazzi, M., Blasques, F., Koopman, S. J., and Lucas, A. (2016). "Time-Varying Transition Probabilities for Markov Regime Switching Models," *Journal of Time Series Analysis* **38**(3), 458–478.

Bekaert, G. and Harvey, C. R. (1995). "Time-Varying World Market Integration," *Journal of Finance* **50**(2), 403–444.

Bengen, W. P. (1994). "Determining Withdrawal Rates Using Historical Data," *Journal of Financial Planning* **7**(4), 14–24.

Bo, L., Wang, Y., and Wang, X. (2010). "An Optimal Portfolio Problem in a Defaultable Market," *Advances in Applied Probability* **42**, 689–705.

Brandt, M. W. (2010). "Portfolio Choice Problems," in Ait-Sahalia, Y. and Hansen, L. P., eds. *Handbook of Financial Econometrics: Tools and Techniques*, North Holland, pp. 269–336.

Brown, K. C. and Tiu, C. I. (2013). "The Interaction of Spending Policies, Asset Allocation Strategies, and Investment Performance at University Endowment Funds," in Brown, K. C., ed. *How the Financial Crisis and Great Recession Affected Higher Education*. Chicago, IL: University of Chicago Press, pp. 43–98.

Brown, S. J., Goetzmann, W. N., and Ross, S. A. (1995). "Survival," *Journal of Finance* **50**(3), 853–873.

Campbell, J. Y., Chan, Y. L., and Viceira, L. M. (2003). "A Multivariate Model of Strategic Asset Allocation," *Journal of Financial Economics* **67**(1), 41–80.

Campbell, J. Y. and Viceira, L. M. (1999). "Consumption and Portfolio Decisions when Expected Returns are Time Varying," *Quarterly Journal of Economics* **114**(3), 433–495.

Cohen, J., Ericson, K. M., Laibson, D., and White, J. M. (2020). "Measuring Time Preferences," *Journal of Economic Literature* **58**(2), 299–347.

Das, S., Ostrov, D., Casanova, A., Radhakrishnan, A., and Srivastav, D. (2022). "Optimal Goals-Based Investment Strategies for Switching between Bull and Bear Markets," *Journal of Wealth Management* **24**(4), 8–36.

Das, S. R. and Uppal, R. (2004). "Systemic Risk and International Portfolio Choice," *Journal of Finance* **59**(6), 2809–2834.

Diebold, F. X., Lee, J.-H., and Weinbach, G. (1999). "Regime Switching with Time-Varying Probabilities," in Diebold, F. X. and Rudebusch, G. D., eds., *Business Cycles: Durations, Dynamics, and Forecasting*, Princeton University Press, pp. 144–166.

Dimson, E., Marsh, P., and Staunton, M. (2021). "American Exceptionalism: The Long-Term Evidence," *Journal of Portfolio Management* **47**(7), 14–26.

Dimson, E., Marsh, P., and Staunton, M. (2023). "Credit Suisse Global Investment Returns Yearbook 2023," Credit Suisse.

Garlappi, L. and Skoulakis, G. (2010). "Solving Consumption and Portfolio Choice Problems: The State Variable Decomposition Method," *Review of Financial Studies* **23**(9), 3346–3400.

Guidolin, M. and Timmermann, A. (2008). "International Asset Allocation under Regime Switching, Skew, and Kurtosis Preferences," *Review of Financial Studies* **21**(2), 889–935.

Hamilton, J. D. (1989). "A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle," *Econometrica* **57**(2), 357–384.

Harris, L. E., Hartzmark, S. M., and Solomon, D. H. (2015). "Juicing the Dividend Yield: Mutual Funds and the Demand for Dividends," *Journal of Financial Economics* **116**(3), 433–451.

He, R., Siwinski, S., Yu, C., and Ang, A. (2022). "Public Pensions in a World of Low Rates and Low Risk Premiums," *Journal of Investment Consulting* **21**(1), 30–44.

Jorion, P. and Goetzmann, W. N. (1999). "Global Stock Markets in the Twentieth Century," *Journal of Finance* **54**(3), 953–980.

Kandel. S. and Stambaugh, R. F. (1996). "On the Predictability of Stock Returns: An Asset Allocation Perspective," *Journal of Finance* **51**(2), 385–424.

Kochenderfer, M. J. and Wheeler, T. A. (2019). *Algorithms for Optimization* (Cambridge, MA: MIT Press).

Lewin, M. and Campani, C. H. 2022. "Constrained Portfolio Strategies in a Regime-Switching Economy," *Financial Markets and Portfolio Management*, https://doi.org/10.1007/s11408-022-00414-x.

Liu, J., Longstaff, F. A., and Pan, J. (2003). "Dynamic Asset Allocation with Event Risk," *Journal of Finance* **58**(1), 231–259.

Merton, R. C. (1969). "Lifetime Portfolio Selection under Uncertainty: The Continuous-Time Case," *Review of Economics and Statistics* **51**(3), 247–257.

Merton, R. C. (1971). "Optimum Consumption and Portfolio Rules in a Continuous-Time Model," *Journal of Economic Theory* **3**(4), 373–413.

Muller, L. (2022). "Optimal Portfolio Choice with Crash and Default Risk," *International Journal of Theoretical and Applied Finance* **25**(4–5), 2250023.

Pratt, J. W. (1964). "Risk Aversion in the Small and in the Large," *Econometrica* **32**(1/2), 122–136.

Samuelson, P. A. (1969). "Lifetime Portfolio Selection by Dynamic Stochastic Programming," *Review of Economics and Statistics* **51**(3), 239–246.

Tu, J. (2010). "Is Regime Switching in Stock Returns Important in Portfolio Decisions?," *Management Science* 56(7), 1198–1215.

Van der Ploeg, F. and Rezai, A. (2020). "Stranded Assets in the Transition to a Carbon-Free Economy," *Annual Review of Resource Economics* **12**, 281–298.

Wachter, J. A. (2002). "Portfolio and Consumption Decisions under Mean-Reverting Returns: An Explicit Solution for Complete Markets," *Journal of Financial and Quantitative Analysis* **37**(1), 63–91.

Wu, L. (2003). "Jumps and Dynamic Asset Allocation," *Review of Quantitative Finance and Accounting* **20**, 207–243.

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