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ON THE USE OF THE DAILY FAMA–FRENCH RISK-FREE RATE

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The Fama and French (1992) risk-free rate is used throughout the extant finance literature. The daily risk-free series has issues that raise concerns about its use as a benchmark. We detail the issues and discuss viable low-cost alternatives. We suggest the use of an adjusted one-year constant maturity rate for empirical analysis dating back to July 1, 1963. Our empirical results suggest that the choice matters in short-run analyses or single-day event studies, but not in studies that employ long-run averages, as is typical in asset-pricing research. We also document an interesting methodological division in empirical academic financial analysis.

1 Introduction

The risk-free rate is an essential variable in many models used throughout the extant finance literature. Sharpe (1964) states that the market provides two prices to investors: (1) the price of time and (2) the price of risk. Sharpe refers to the price of time as the pure interest rate. Over time, this interest rate has become known as the risk-free rate. The risk-free rate is often a presumed variable, and a standard proxy is the Fama–French risk-free rate (henceforth, FFRF). The purpose of this paper is to examine the methodology used to construct the FFRF and to provide a more accurate estimate of the risk-free rate for future academic research.

Our investigation into the utilization of the FFRF stems from a statement in the article titled *Stock Returns over the FOMC Cycle* by Cieslak *et al.* (working paper, 2016).[1] In Section 2.A. Main Result (p. 5), the authors describe their methodology as:

"We graph the 5-day cumulative stock return returns from (and including) day t to day $t+4$ minus the 5-day cumulative return on 30-day Treasury bills from day t to day $t + 4$."

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This statement caught our attention because a "30-day"[2] Treasury bill (T-bill) does not exist and it includes the following footnote: "For ease of replicating the result, we use stock returns and T-bill returns from the Fama–French factor files on Ken French's website." We appreciate the authors thinking about replications, and the Fama–French data make much sense here: (1) it is ubiquitously used in the literature, (2) it is easy to access, and (3) it is free.[3] Ken French's data site[4] describes the T-bill as a one-month bill. The term "one-month" is a better and more familiar description than "30-days," but it is also inaccurate.[5] The correct description for the "one-month" bill is a four-week bill, which typically has 28-days to maturity.[6] A four-week bill rarely covers exactly one month and never has 30-days to maturity.[7] These varying descriptions of the risk-free rate inspired a more in-depth look into the calculation of the risk-free rate benchmark. Accordingly, we examine the assumptions made in calculating the daily Fama–French risk-free rate and the potential errors, if any, that arise from the underlying construction of the FFRF.

We show that the methods underlying the daily Fama–French risk-free rate have empirical issues when viewed from a contemporary perspective. Specifically, the daily FFRF is constant across all trading days in a month. The FFRF spreads weekend returns across all trading days in a month, and the rate resets at the start of each month. Thus, these data are not reasonable proxies for daily market data. We believe that the availability of improved data and methods justifies updating the daily FFRF as the marginal cost for implementing a new measure is low.

Our empirical analysis suggests that the difference between the daily FFRF and a non-seasonally adjusted, constant maturity rate (CMTBR) is minor when long-run averages are involved. We replicate a table from Fama and French (1992) and do not find a statistically significant difference in the coefficient estimates of the three-factor model for the two risk-free rate series. Our results suggest that over a long period, the updating of the FFRF is not statistically significant. Next, we replicate Cieslak *et al.* (2016). Cieslak *et al.* (2016) examine excess returns post Federal Open Market Committee (FOMC) meetings, and they find statistically significant excess return estimates. Our results confirm their findings. Again, we do not find a statistical difference between the FFRF and the CMTBR. We believe the non-result stems from the averaging of excess returns over 20 years.

We take our analysis a step further and focus on one year of the excess returns post FOMC meetings. At the extreme, we find a 5.6 basis points per week difference, which annualizes to 2.91%. We also create a $1 million long/short buy-and-hold portfolio strategy on the same year of data and find an arbitrage profit of $138,868. These results suggest significant economic opportunities. We conclude that without long-run averaging, choosing between the daily FFRF and CMTBR can result in significant differences. With our findings, we review the literature for essential papers that use the FFRF in situations where its use might alter the interpretation of the results. We did not find any papers exhibiting this problem. Instead, we find a critical methodological difference in the choice of the risk-free rate across the finance literature with asset-pricing and long-run analyses using FFRF and with money market analyses using better proxies for the risk-free rate.

We examine a variety of different risk-free rates that could be used to modernize the FFRF data and conclude that researchers should use the one-month CMTBR data for studies beginning in July 2001. We recommend using the non-seasonally adjusted, one-month, constant maturity rate (CMTBR) provided by the Federal

Reserve Economic Database (FRED).[8] We do, however, make one modification to the data. That is, we adjust the CMTBR to account for non-trading days. While qualitatively similar, the weekend-adjusted CMTBR is a better measure of the market's interpretation of the risk-free rate.

The rest of this paper is organized as follows. Section 2 describes the methodology behind the derivation of the daily Fama–French risk-free return, and we discuss the unintended pitfalls. In Section 3, we explore alternative measures and make our recommendation to update the FFRF. Section 4 discusses the type of research where using traditional data may lead to significant differences in results. We conclude by discussing a serious issue within the finance literature, which supports our recommendation for updating the FFRF.

2 Understanding the Fama–French Risk-Free Returns

According to Ken French's webpage, the construction of the daily risk-free rate derives from the following[9]:

"The T-bill return is the simple daily rate that, over the number of trading days in the month, compounds to 1-month T-Bill rate from Ibbotson and Associates, Inc."

Therefore, our next step is to determine how Ibbotson calculates the one-month T-bill rate. We contacted Morningstar/Ibbotson[10] and spoke with a data analyst to verify their process for calculating the risk-free returns.[11] On the last trading day of the month, they select the T-bill closest to 30 days to maturity but not less than 30 days and collect the bid/ask quote midpoint as the price. Then, the return over the following month assumes this bill matures on the last trading day of the month.[12] This method follows Fama (1975) where Equation (1) provides the return on a T-bill with one month to maturity at $t-1$ as:

$$R_t = \frac{V_t - V_{t-1}}{V_{t-1}} = \frac{\$1{,}000 - V_{t-1}}{V_{t-1}}. \quad (1)$$

The calculation shown in Equation (1) is a simple monthly return, but with two interesting assumptions. The first assumption is that $V_t = \$1{,}000$, which assumes that each T-bill matures on the last trading day of the month, which is only possible if the last trading in the month is a Thursday.[13] The second assumption is that V_{t-1} is for the bill with the closest to 30 days to maturity, but with at least 30 days to maturity. This bill is unlikely to have exactly 30 days to maturity, and when it does, it does not always mature on the last trading day of the next month.

We use the perpetual calendar to create all the possible month-end pairs and report the combinations using the Ibbotson definition. Table 1 presents the month-end pairs.

The perpetual calendar has 14 possible year variations, and with 12 months per year, there are 168 possible month-end pairs. The first number in each cell of Table 1 is the number of days to maturity for the T-bill closest to 30 days to maturity with at least 30 days to maturity. The second number in each cell is the number of days between the last trading days for each month-end pair. The second number is missing from the December column because the following January is year specific.

In Table 1, we find eight monthly combinations (4.76%) having a 30-day T-bill that matures on the last trading day of the next month. Second, if we generalize the definition to a "one-month" return that matures on the last trading day of the next month, which would include 31-day bills, we find an additional 12 cells. In total, we find that only 20 one-month pairs out of 168 cells (11.90%) satisfy this definition. This result implies that, at best, the return calculation used in the Ibbotson

Table 1 Fama–French T-bill Days.

	Jan	Feb	March	April	May	June	July	Aug	Sept	Oct	Nov	Dec
1	30/28	30/31	34/28	34/33	36/30	34/31	*31/31*	35/29	34/32	**30/30**	35/29	34/na
2	36/28	36/30	34/31	*31/31*	35/29	34/32	30/31	34/28	34/33	36/30	34/31	31/na
3	35/28	35/29	34/32	30/31	34/28	34/33	36/30	34/31	31/32	35/29	34/33	30/na
4	34/28	34/31	31/30	36/30	34/30	*31/31*	35/29	34/32	37/31	34/28	34/33	36/na
5	34/28	34/32	**30/30**	35/29	34/32	30/31	34/31	31/30	36/30	34/31	*31/31*	35/na
6	34/28	34/33	36/30	34/31	34/33	36/30	34/32	**30/30**	35/29	34/32	30/31	34/na
7	31/28	*31/31*	35/29	34/32	**30/30**	35/29	34/33	36/30	34/31	31/30	36/30	34/na
8	30/29	36/30	34/31	*31/31*	35/29	34/32	*31/31*	34/28	34/33	36/30	34/31	31/na
9	36/29	35/29	34/32	30/31	34/28	34/33	36/30	34/30	*31/31*	35/29	34/32	31/na
10	35/29	34/31	31/30	36/30	34/31	*31/31*	35/29	34/32	30/31	34/28	34/33	36/na
11	34/28	34/32	**30/30**	35/29	34/32	30/31	34/31	31/30	36/30	34/31	*31/31*	35/na
12	34/28	34/33	36/30	34/31	31/30	36/30	34/32	**30/30**	35/29	34/32	30/31	34/na
13	34/31	*31/31*	35/29	34/32	**30/30**	35/29	34/33	36/30	34/31	31/30	36/30	34/na
14	31/29	30/31	34/28	34/33	36/30	34/31	*31/31*	35/29	34/33	**30/30**	35/29	34/na

The perpetual calendar has 14 different year patterns. This table takes each year's pattern and presents the number of calendar days for the T-bills used in Fama–French (1992) to calculate one-month T-bill returns. The first number in each cell is the number of days to maturity for the T-bill closest to 30 days to maturity with at least 30 days to maturity. The second number is the number of days from the last trading day of the month until the last trading day of the next month. There are 168 cells in this table, and only eight cells (4.76%) have a 30-day T-bill with 30-days until the last trading day of the next month.

data and reported in the daily Fama–French data is correct less than 12% of the time.[14]

Recall that the Fama (1975) method assumes the bill matures and pays $1,000. Table 1 shows 109 of the 168 (64.88%) cells have T-bills that have 34, 35, or 36 days to maturity at $t-1$. Since T-bills are discount securities, the price is lower the farther the security is from maturity. Accordingly, in almost 65% of the available months, the Fama (1975) method would include between four and six days of extra return. In an excess return calculation on a stock, the stock has only one month to earn the return while the reference T-bill has one month plus N days to earn its return, which should bias the excess return downward.

Next, we examine the assumption for the purchase price, V_{t-1}, of the one-month T-bill in the Fama (1975). According to Ibbotson, the price represents the midpoint of the bid and ask quotes, which is not the price at which the bill could be purchased. Thus, we calculate monthly T-bill returns for buying the bill at the ask price at $t-1$ and selling the T-bill at the bid price at t. Using data from T-bill trades reported in GovPX during 1997, we calculate the actual monthly trading returns. We compare these one-month returns with the one-month returns for the FFRF data and find an average difference of 0.41 basis points with actual returns higher than FFRF. Actual returns exceed FFRF in 10 of 11 months (we do not have December 1996 data, so we cannot calculate January returns). With FFRF being less than actual trading return, the excess returns are overstated when FFRF returns are used relative to actual returns.

In Table 2, we report the daily FFRF to illustrate the implications of the methodology. For the sake of brevity, we report the daily FFRF for March in the years: 1954, 1964, 1974, 1984, 1994, 2004, and 2014.[15]

Table 2 Fama–French Daily Risk-Free Rate.

March	1954	1964	1974	1984	1994	2004	2014
1	0.003		0.026	0.033	0.012	0.004	
2	0.003	0.015		0.033	0.012	0.004	
3	0.003	0.015			0.012	0.004	0.000
4	0.003	0.015	0.026		0.012	0.004	0.000
5	0.003	0.015	0.026	0.033		0.004	0.000
6		0.015	0.026	0.033			0.000
7			0.026	0.033	0.012		0.000
8	0.003		0.026	0.033	0.012	0.004	
9	0.003	0.015		0.033	0.012	0.004	
10	0.003	0.015			0.012	0.004	0.000
11	0.003	0.015	0.026		0.012	0.004	0.000
12	0.003	0.015	0.026	0.033		0.004	0.000
13		0.015	0.026	0.033			0.000
14			0.026	0.033	0.012		0.000
15	0.003		0.026	0.033	0.012	0.004	
16	0.003	0.015		0.033	0.012	0.004	
17	0.003	0.015			0.012	0.004	0.000
18	0.003	0.015	0.026		0.012	0.004	0.000
19	0.003	0.015	0.026	0.033		0.004	0.000
20		0.015	0.026	0.033			0.000
21			0.026	0.033	0.012		0.000
22	0.003		0.026	0.033	0.012	0.004	
23	0.003	0.015		0.033	0.012	0.004	
24	0.003	0.015			0.012	0.004	0.000
25	0.003	0.015	0.026		0.012	0.004	0.000
26	0.003	0.015	0.026	0.033		0.004	0.000
27			0.026	0.033			0.000
28			0.026	0.033	0.012		0.000
29	0.003		0.026	0.033	0.012	0.004	
30	0.003	0.015		0.033	0.012	0.004	
31	0.003	0.015			0.012	0.004	0.000

This table presents the daily risk-free rates collected from Ken French's website. This rate is based on the one-month return for a T-bill with 30 or more days to maturity. We arbitrarily selected the month of March and the years 1954, 1964, 1974, 1984, 1994, 2004, and 2014 because the pattern within each month is representative of the entire data set. The risk-free rates in this table are in percentages. For example, in 1984, the monthly rate equals 0.033% times 22 trading days, or 0.76% monthly rate, which is equivalent to a 9.12% annualized rate.

Table 2 shows that within each month, the daily T-bill return is the same every day. These risk-free returns are not consistent with daily market data as T-bill rates change on a daily basis. Instead, the constant daily risk-free returns are algorithmically generated with a monthly jump process; that is, the daily rates update on the first trading day of each month.

One additional point needs to be made from the data found in Table 2. The FFRF calculation does not consider the weekend effect. That is, monthly T-bill returns are spread evenly across trading days without accounting for the fact that the price of a discount instrument changes (accretes) every day, regardless of trading as the instrument moves toward maturity and the price moves toward the face value. Accordingly, the proper approach to spreading monthly T-bill returns across days is to divide by calendar days, not trading days, and assign Mondays three days of returns to account for Saturday and Sunday.[16] In contrast, stock prices are only updated when a trade is made (or when the bid–ask spread is adjusted and the mean is used in the event of no trade), and since markets are closed on the weekends, the return calculation for stocks from Friday's close to Monday's close accounts for the weekend. We believe the risk-free rate should be adjusted to account for the weekend. In the next section, we provide a recommendation for updating the FFRF.

3 Updating the Fama–French Risk-Free Rate

The risk-free rate is the shortest-term default-free security. Fama and French (1992) use an approximate one-month T-bill (FFRF), which comes in the form of an off-the-run three-month bill with roughly one month to maturity. In the prior section, we showed the various short-comings with that choice. In this section, we offer an alternative to update the data.

The best substitute for the daily FFRF would be the daily rate from a one-month T-bill. Unfortunately, the U.S. Treasury does not issue a one-month T-bill. It does issue a four-week T-bill, but this did not start until July of 2001. The four-week data series is not long enough to fit the needs of researchers analyzing earlier periods.[17] Since the FFRF goes back to July of 1963, we want to provide a replacement for future research that goes back to that date.

The Federal Reserve Economic Data (FRED) provides a constant maturity series which contains a time series of daily risk-free rates that address the problems of the secondary market data.[18,19] The constant maturity daily series for bills begin (1) 1/4/82 for three-month bills, (2) 1/4/82 for six-month bills, and (3) 1/25/62 for one-year bills. The daily constant maturity series for one-year bills (CMTBR) is the only series that begins early enough to work with data going back to July of 1963. Accordingly, the remainder of this section examines the viability of the one-year CMTBR as the benchmark risk-free rate for future research.

We collect the daily one-year constant maturity series from FRED from July 1, 1963 to December 31, 2018. Figure 1 plots the time series of the two data series with the CMTBR divided by 365 to convert the annualized rate into a daily rate. The figure shows that the FFRF tends to be higher than the CMTBR for most of the sample. This result is surprising given that the short-end of the term structure is typically upward sloping.

To compare these rates, Figure 2 plots the spread between the two rates, where the spread equals the CMTBR minus the FFRF.

We observe that the majority of these differences are negative, indicating that the FFRF is higher than the CMTBR. Additionally, we see that the vast majority of the sample where the FFRF is less

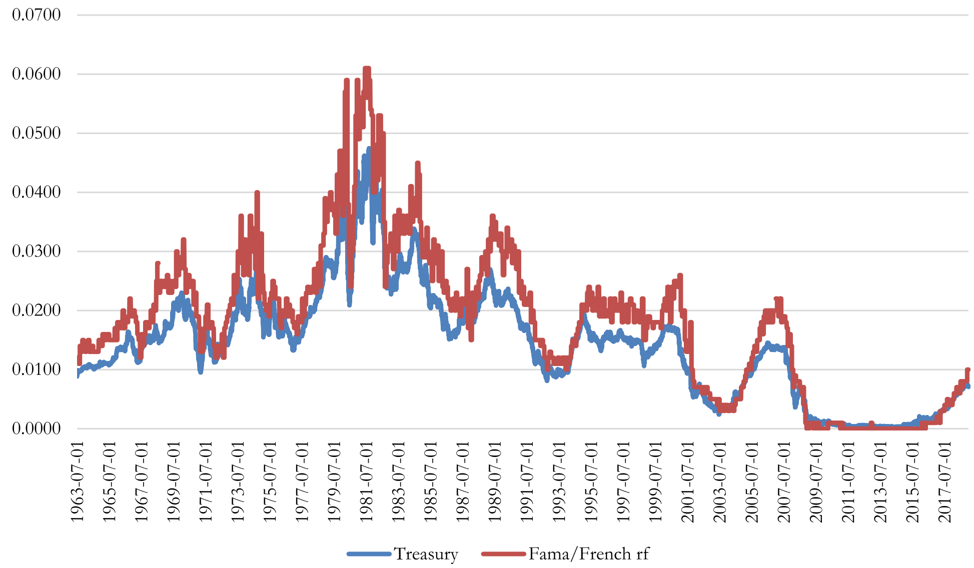

Figure 1 Daily risk-free rate proxies.

This figure plots the one-year constant maturity yield (dark gray) versus the Fama–French risk-free rate (light gray) from July 1, 1963 to December 31, 2017. The figure shows that the daily Fama–French risk-free rate tends to be higher than the daily one-year constant maturity yield.

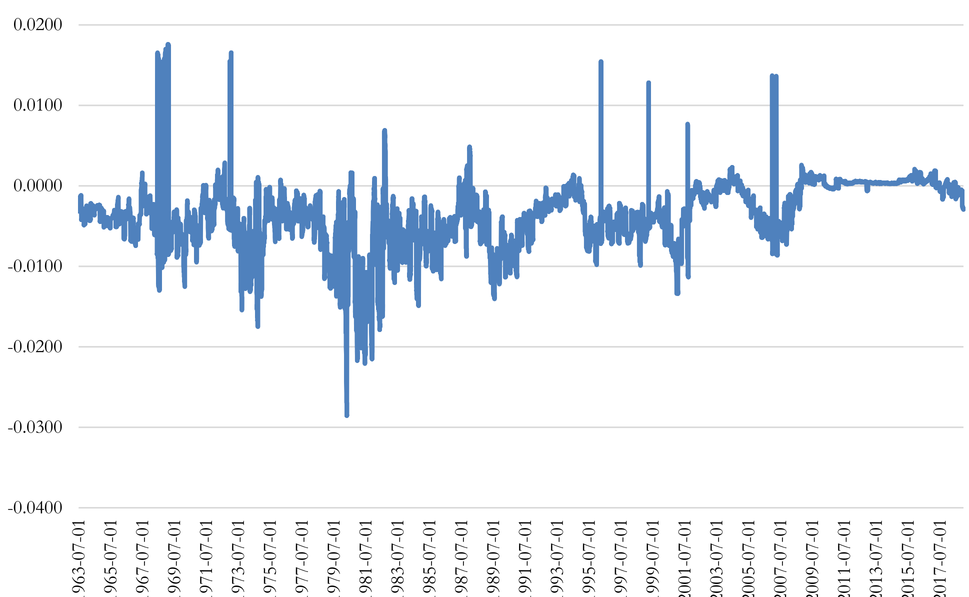

Figure 2 Daily one-year constant maturity rate minus daily Fama–French risk-free rate.

This figure illustrates the difference between the daily one-year constant maturity rate and the daily Fama–French rate from July 1, 1963 to December 31, 2017. The majority of these differences are negative (80.7%), which indicates that the daily Fama–French risk-free rate is higher than the daily one-year constant maturity rate.

Table 3 Daily Fama–French and One-Year Constant Maturity Rate.

	Fama–French Risk-Free Rate						One-Year Constant Maturity Rate					
	1964	1974	1984	1994	2004	2014	1964	1974	1984	1994	2004	2014
1		0.026	0.033	0.012	0.004			0.0198	0.0282	0.0114	0.0034	
2	0.015		0.033	0.012	0.004		0.0107		0.0279	0.0114	0.0035	
3	0.015			0.012	0.004	0.000	0.0107			0.0115	0.0035	0.0003
4	0.015	0.026		0.012	0.004	0.000	0.0107	0.0199		0.0117	0.0034	0.0003
5	0.015	0.026	0.033		0.004	0.000	0.0107	0.0200	0.0281		0.0032	0.0004
6	0.015	0.026	0.033			0.000	0.0107	0.0199	0.0281			0.0003
7		0.026	0.033	0.012		0.000		0.0199	0.0283	0.0116		0.0004
8		0.026	0.033	0.012	0.004			0.0203	0.0284	0.0117	0.0032	
9	0.015		0.033	0.012	0.004		0.0107		0.0287	0.0118	0.0032	
10	0.015			0.012	0.004	0.000	0.0107			0.0118	0.0032	0.0003
11	0.015	0.026		0.012	0.004	0.000	0.0107	0.0200		0.0117	0.0032	0.0004
12	0.015	0.026	0.033		0.004	0.000	0.0107	0.0202	0.0286		0.0032	0.0003
13	0.015	0.026	0.033			0.000	0.0107	0.0205	0.0287			0.0003
14		0.026	0.033	0.012		0.000		0.0204	0.0289	0.0118		0.0003
15		0.026	0.033	0.012	0.004			0.0207	0.0288	0.0118	0.0032	
16	0.015		0.033	0.012	0.004		0.0107		0.0292	0.0117	0.0032	
17	0.015			0.012	0.004	0.000	0.0107			0.0117	0.0032	0.0004
18	0.015	0.026		0.012	0.004	0.000	0.0107	0.0211		0.0119	0.0032	0.0004
19	0.015	0.026	0.033		0.004	0.000	0.0107	0.0213	0.0294		0.0033	0.0004
20	0.015	0.026	0.033			0.000	0.0107	0.0219	0.0296			0.0004
21		0.026	0.033	0.012		0.000		0.0230	0.0297	0.0121		0.0004
22		0.026	0.033	0.012	0.004			0.0232	0.0299	0.0119	0.0032	
23	0.015		0.033	0.012	0.004		0.0107		0.0299	0.0118	0.0032	
24	0.015			0.012	0.004	0.000	0.0108			0.0120	0.0032	0.0004
25	0.015	0.026		0.012	0.004	0.000	0.0108	0.0232		0.0120	0.0032	0.0004
26	0.015	0.026	0.033		0.004	0.000	0.0108	0.0229	0.0298		0.0033	0.0003
27		0.026	0.033			0.000		0.0224	0.0297			0.0003
28		0.026	0.033	0.012		0.000		0.0232	0.0295	0.0121		0.0004
29		0.026	0.033	0.012	0.004			0.0229	0.0294	0.0122	0.0033	
30	0.015		0.033	0.012	0.004		0.0109		0.0296	0.0122	0.0033	
31	0.015			0.012	0.004	0.000	0.0108			0.0122	0.0033	0.0004

The left-hand side is from Table 1. We append the daily one-year constant maturity rates for comparison on the same dates for March in 1964, 1974, 1984, 1994, 2004, and 2014. We exclude 1954 from the table because the one-year constant maturity started in July 1963. This table shows that the daily one-year constant maturity rates are not the same each day. The risk-free rates in this table are in percentages.

Table 4 Daily Fama–French and Weekend-Adjusted One-Year Constant Maturity Rate.

	Fama–French Risk-Free Rate						One-Year Constant Maturity Rate					
	1964	1974	1984	1994	2004	2014	1964	1974	1984	1994	2004	2014
1		0.026	0.033	0.012	0.004			0.0198	0.0282	0.0114	0.0100	
2	0.015		0.033	0.012	0.004		0.0321		0.0279	0.0114	0.0035	
3	0.015			0.012	0.004	0.000	0.0107			0.0115	0.0035	0.0010
4	0.015	0.026		0.012	0.004	0.000	0.0107	0.0596		0.0117	0.0034	0.0003
5	0.015	0.026	0.033		0.004	0.000	0.0107	0.0200	0.0838		0.0032	0.0004
6	0.015	0.026	0.033			0.000	0.0107	0.0199	0.0281			0.0003
7		0.026	0.033	0.012		0.000		0.0199	0.0283	0.0349		0.0004
8		0.026	0.033	0.012	0.004			0.0203	0.0284	0.0117	0.0095	
9	0.015		0.033	0.012	0.004		0.0321		0.0287	0.0118	0.0032	
10	0.015			0.012	0.004	0.000	0.0107			0.0118	0.0032	0.0010
11	0.015	0.026		0.012	0.004	0.000	0.0107	0.0605		0.0117	0.0032	0.0004
12	0.015	0.026	0.033		0.004	0.000	0.0107	0.0202	0.0259		0.0032	0.0003
13	0.015	0.026	0.033			0.000	0.0107	0.0205	0.0287			0.0003
14		0.026	0.033	0.012		0.000		0.0204	0.0289	0.0353		0.0003
15		0.026	0.033	0.012	0.004			0.0207	0.0288	0.0118	0.0097	
16	0.015		0.033	0.012	0.004		0.0320		0.0292	0.0117	0.0032	
17	0.015			0.012	0.004	0.000	0.0107			0.0117	0.0032	0.0010
18	0.015	0.026		0.012	0.004	0.000	0.0107	0.0624		0.0119	0.0032	0.0004
19	0.015	0.026	0.033		0.004	0.000	0.0107	0.0213	0.0878		0.0033	0.0004
20	0.015	0.026	0.033			0.000	0.0107	0.0219	0.0296			0.0004
21		0.026	0.033	0.012		0.000		0.0230	0.0297	0.0359		0.0004
22		0.026	0.033	0.012	0.004			0.0232	0.0299	0.0119	0.0097	
23	0.015		0.033	0.012	0.004		0.0320		0.0299	0.0118	0.0032	
24	0.015			0.012	0.004	0.000	0.0108			0.0120	0.0032	0.0012
25	0.015	0.026		0.012	0.004	0.000	0.0108	0.0696		0.0120	0.0032	0.0004
26	0.015	0.026	0.033		0.004	0.000	0.0108	0.0229	0.0896		0.0033	0.0003
27		0.026	0.033			0.000		0.0224	0.0297			0.0003
28		0.026	0.033	0.012		0.000		0.0232	0.0295	0.0361		0.0004
29		0.026	0.033	0.012	0.004			0.0229	0.0294	0.0122	0.0098	
30	0.015		0.033	0.012	0.004		0.0433		0.0296	0.0122	0.0033	
31	0.015			0.012	0.004	0.000	0.0108			0.0122	0.0033	0.0011
Total	0.315	0.546	0.726	0.276	0.092	0.000	0.343	0.614	0.810	0.367	0.107	0.011

This table is identical to Table 3, except for the fact that we adjusted the daily one-year constant maturity rates for weekends. Notice that the Monday value is much more significant in the daily one-year constant maturity rate columns. The risk-free rates in this table are in percentages.

than the CMTBR occurs following the financial crisis when the FFRF equals 0% each day.

In Table 3, we replicate Table 2 and add the CMTBR without 1954.

Table 3 illustrates the fundamental difference between the two risk-free time series. The daily FFRF is constant throughout each month (by construction), whereas the CMTBR updates daily. Thus, the CMTBR is more indicative of an actual market risk-free rate.

Finding that the daily FFRF is higher than the daily CMTBR is surprising, in theory, because one security is based on a three-month bill, and the other assumes one-year to maturity. However, there are two parts to creating FFRF that offer an explanation. First, FFRF uses a T-bill with 30 or more days to maturity. These extra days increase yield. Second, FFRF is calculated as a monthly yield and then spread evenly across the trading days in a month (see Table 3). In Table 3 and Figures 1 and 2, we do not attempt to adjust for weekends in the constant maturity rate series. Instead, to this point, we simply assign a one-day rate to each trading day.

The price of time includes Saturday and Sunday. The FFRF includes the weekends by spreading the monthly yield evenly across all trading days. The correct adjustment for non-trading days in the daily CMTBR is to add the rate of the non-trading days to the next trading day, where the rate for fixed income securities over a non-trading day is the rate on the previous trading day. Accordingly, we make this adjustment to the CMTBR. Table 4 replicates Table 3 with the adjustment for non-trading days; this data series is our proposed better measure. We also add a row to the bottom of the table for total daily interest across the month. Table 4 shows higher interest rates on the days following non-trading days, which is consistent with daily stock returns on Monday being calculated from the Friday close to the Monday close. Also,

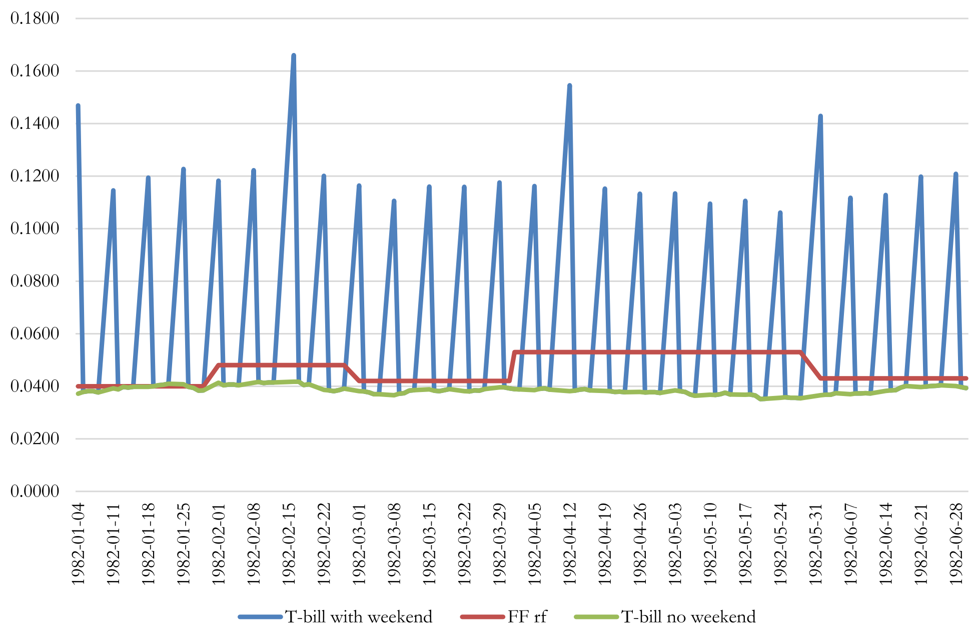

Figure 3A Risk-free rate comparison.

This figure presents a time series plot of the different risk-free rates from January 1, 1982 to June 30, 1982. The line at the very bottom represents the daily one-year constant maturity rate. The spiked line is the one-year constant maturity rate adjusted for weekends, and the stair-step line represents the daily Fama–French risk-free rate, which only changes at the beginning of each month.

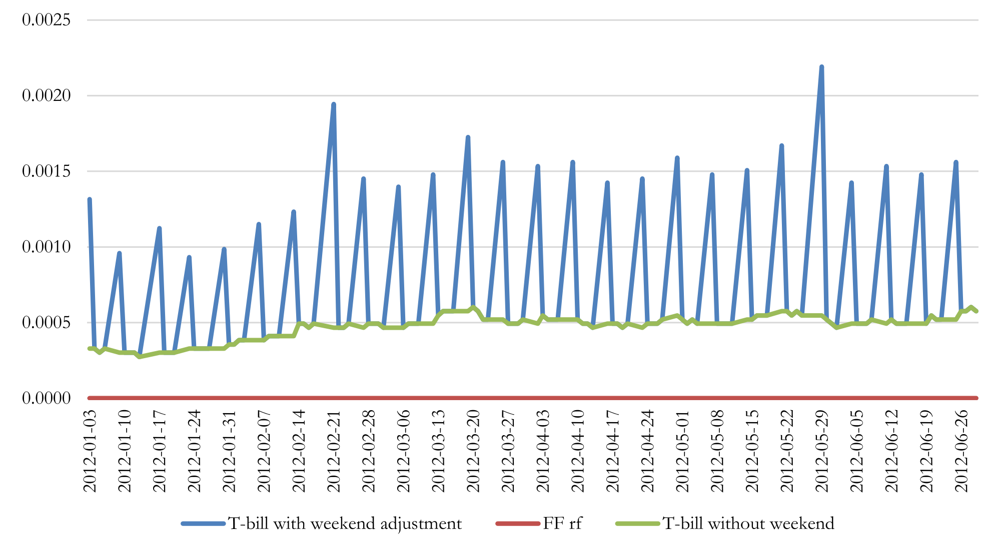

Figure 3B Risk-free rate comparison.

This figure presents a time series plot of the different risk-free rates from January 1, 2012, to June 30, 2012. The line at the very bottom represents the Fama–French risk-free rate, which equals 0 every day. The spiked line is the daily one-year constant maturity rate adjusted for weekends, and the line below the spikes represent the daily one-year constant maturity rate.

after including the interest from non-trading days, the total monthly interest from CMTBR exceeds the total monthly interest from the FFRF, which is consistent with an upward sloping yield curve.

We would like to replicate Figure 1 using our weekend adjusted CMTBR. However, weekly spikes in a plot of daily rates over a sample period of 54½years make the figure unreadable. Therefore, in an attempt to provide insight into the different data series, we plot the first six months of 1982 and 2012 in Figure 3. We chose 1982 because it was a period of historically high rates, and we chose 2012 because it was a period of the historically low rates.

Figure 3A shows that the FFRF is constant each day throughout the month, and the CMTBR tends to be lower; however, the weekend adjusted CMTBR exhibits spikes above both series. Figure 3B compares the three series when interest rates were historically low (specifically 2012, generally post-financial crisis). In Figure 3B, we see that the FFRF is lower than the CMTBR. Both plots show some variability in the CMTBR, but no variability during any month of the FFRF. The plots of the daily FFRF do not resemble market data. Admittedly, the weekend adjusted CMTBR does not look like market data because of the spikes on Mondays to reflect the accumulation of interest over the weekend. However, the price of time is continuous, while markets close periodically. With daily data, an investor can capture the weekend interest on Monday, which leads to the Monday spikes in our recommended series.

4 Supporting Evidence

We believe that it is vital for researchers to understand the implications that the daily FFRF series might have on their empirical findings. In this section, we ask the question: "Does the daily CMTBR produce different empirical results? We answer this question by first presenting the types of studies where updating the FFRF is least likely to matter statistically and follow with a discussion of where there could be discrepancies in results.

Fama–French (1992) had over 19,000 citations as of 2018. Thus, it would be an exhaustive process to replicate all of the empirical results to date or an impossible task to pick one best paper to replicate. We do not attempt to do either. Instead, we re-examine the two papers that drive our analysis: (1) Fama and French (1992) and Cieslak *et al.* (2016). Fama and French (1992) is a time series asset-pricing paper, while Cieslak *et al.*'s (2016) paper is a short-window event study over a long period. Both use the daily FFRF as the benchmark risk-free rate when calculating excess returns.

We replicate Fama and French Table 4 (1992, p. 448) using the daily Fama–French three-factor data with two key differences: the period is extended, and we do not report the same sub-periods.[20] We collect two daily return series from CRSP: the equally weighted index with dividends and the value-weighted index with dividends from July 1, 1963 to December 31, 2018. We collect the three factors: Mkt-Rf, SMB, HML, and FFRF from Ken French's website. For comparison, we estimate the model using the two different risk-free rates (FFRF and the weekend adjusted CMTBR), which creates two different market risk premium measures. To create the second market risk premium, we add the FFRF back into Mkt-Rf and then subtract the weekend adjusted CMTBR. The dependent variable is the excess returns on the CRSP index. We report our findings in Table 5.

Table 5 Fama–French Three-Factor Estimation.

	Fama–French Risk-Free Rate	One-Year T-Bill Risk-Free Rate
Panel A: Equally weighted CRSP excess return		
Intercept	0.0298	0.0292
Mkt-Rf	0.7929	0.7934
SMB	0.6271	0.6276
HML	0.0853	0.0853
Panel B: Value-weighted CRSP excess return		
Intercept	−0.0009	−0.0009
Mkt-Rf	0.9899	0.9899
SMB	0.0131	0.0131
HML	0.0097	0.0097

This table reports the coefficients on the three-factor model from July 1, 1963 to December 31, 2018. Panel A reports the coefficient estimates using the equal-weighted index (with dividends) from CRSP. The first set of coefficients are estimated using the Fama–French risk-free rate and market risk premium. The second set of coefficients are the estimates using our suggested alternative measure—the weekend adjusted daily one-year T-bill risk-free rate. Panel B is identical, except we use the value-weighted index (with dividends) from CRSP as the dependent variable. The table shows that the point estimates every similar using either risk-free rate series.

Panel A reports the coefficient estimates from the three-factor model using the equally weighted excess return series. The coefficient estimates are qualitatively similar, as expected. Differences only appear in the third and fourth decimal points. Panel B presents the coefficient estimates using the value-weighted excess returns. We find point estimates to be the same out to the fourth decimal place. The empirical results are similar to either risk-free rate. Fama–French (1992) study the cross-sectional variation of returns over a long period: July 1963 through December 1990. From our findings, we conclude that the considerable averaging of results over the entire sample reduces the statistical difference in coefficient estimates between the two risk-free rate series.

Cieslak *et al.* (2016) examine stock returns relative to FOMC meeting dates from 1994 through 2015. They define the FOMC meeting date as day 0 in their event process and calculate excess returns for the meeting week (day −1 to day 3) of 0.57%. They also report week 2 (days 9–13) excess returns of 0.30% and week 4 (days 19–23) excess returns of 0.45%. We replicate their results for weeks 2 and 4 using both risk-free rate series. Our results suggest that the Cieslak *et al.* (2016) excess returns are overstated by between ½and 1 basis point.[21] The choice of the risk-free rate does not change the economic or statistical conclusions from the Cieslak *et al.*'s analysis. Once again, our findings suggest that updating the risk-free rate might not matter in this type of study because of the averaging of excess returns over a long period.

We believe that revising the risk-free rate would matter the most in a single-event study, such as when a regulation changes, or event studies based upon a limited number of trading days. To demonstrate this point, we revisit the Cieslak *et al.*'s (2016) study.[22] Rather than averaging the excess returns over the entire period, we calculate the difference between the excess returns using daily weekend adjusted CMTBR and the daily FFRF for the second week after the FOMC meetings in 1995. The weekly differences between the two time series for each of the eight FOMC meetings are presented in Table 6 (in percent):

We observe several sign changes in the spreads. After the third FOMC meeting, we find a difference of 5.6 basis points per week (2.91% per annum) between the two series. The average difference across the eight weeks is 2.18 basis points, which annualizes to 1.13%. Both of the annualized differences are economically significant. This result suggests that utilizing daily FFRF can cause statistical differences in specific empirical settings.

Table 6 Difference in Excess Returns for a Single Year.

Meeting #	FOMC Meetings	Spreads (%)
1	Jan 31–Feb 1	0.0390
2	March 28	0.0407
3	May 23	0.0563
4	July 5–6	0.0061
5	Aug 22	−0.0052
6	Sept 26	0.0443
7	Nov 15	0.0023
8	Dec 19	−0.0095
	Average	0.0218
	Median	0.0226

This table presents the results of the weekly differences between the daily CMTBR and the daily FFRF between the two series for each of the eight Federal Open Market Committee Meetings for 1995. The spreads are the difference between the results of the two series, in percentages.

Continuing our analysis, we implement a long/short portfolio approach to compare the differences in rates from an alternative perspective. To construct the portfolio, we borrow $1 million at the daily FFRF and lend the $1 million at the daily CMTBR.[23] We take this position on each day over the eight weeks (40 trading days) analyzed in Table 6. We assume that we hold each position for one month (to maturity). The total profit from these long/short positions equals $21,364, which annualizes to $138,868. Assuming we can lend and borrow at this implied rate, the results suggest that the difference between the two risk-free rate series represents an arbitrage opportunity. Thus, we conclude from this analysis that an update to the FFRF is warranted when studies rely on specific dates and/or small sample periods. In such cases, the conclusions of any such study and the implications for further research or policy formulation are subject to doubt.

5 Looking to the Future

In Section 3, we select the one-year CMTBR as our daily benchmark rate because it is available back to 1/25/62, which aligns well with the asset-pricing data going back to July of 1963 and find differences between the one-year CMTBR and the FFRF. We show that the differences average out in long-horizon studies, but that the differences potentially matter in short-horizon or single-event studies. These studies are less likely to need to reach back to the 1960s. Thus, in this section, we discuss and examine the best available proxy for daily returns on a one-month risk-free security.

In July of 2001, the Treasury began issuing a new Treasury bill that is frequently referred to as a one-month T-bill. However, the correct description is a four-week T-bill, which is typically issued with 28 days to maturity. The four-week bill seldom covers an entire month and never has 30 days to maturity, which makes secondary market data on the four-week bill an incorrect proxy for a one-month risk-free rate.[24] However, recall from Footnote 19 that the Treasury Department derives constant maturity rates across the term structure. In this process, the Treasury derives a constant maturity one-month T-bill rate. This rate is the best available data series for a one-month risk-free rate, and in this section, we compare the daily constant maturity one-month T-bills rates to daily FFRF.

Figure 4 provides a plot from July 31, 2001 through the end of our sample (December 31, 2018) of the daily rate for FFRF and one-month CMTBR. The plot is in basis points and provides useful insights. We follow the process used for Figure 1, so Figure 4 does not include the adjustment for weekends. The daily rates from the two series follow a similar pattern with the one-month CMTBR lying below the FFRF before the financial crisis and in the last two years of the sample. This relationship is consistent with the relationship between one-year CMTBR and FFRF in Figure 1. The most significant difference between the two rates is about 0.8 basis points, which occurs during the year leading up to the start of the financial crisis in August 2007. So, the one-month CMTBR is unlikely to change results relative to the FFRF in a long-horizon study.

Next, we replicate Table 4 using the one-month CMTBR and report the results in Table 7. Table 7 shows the same essential results as Table 4 with FFRF constant across the trading days of the month, while one-month CMTBR varies across the trading days of the month with jumps on Mondays to represent returns across the weekends.

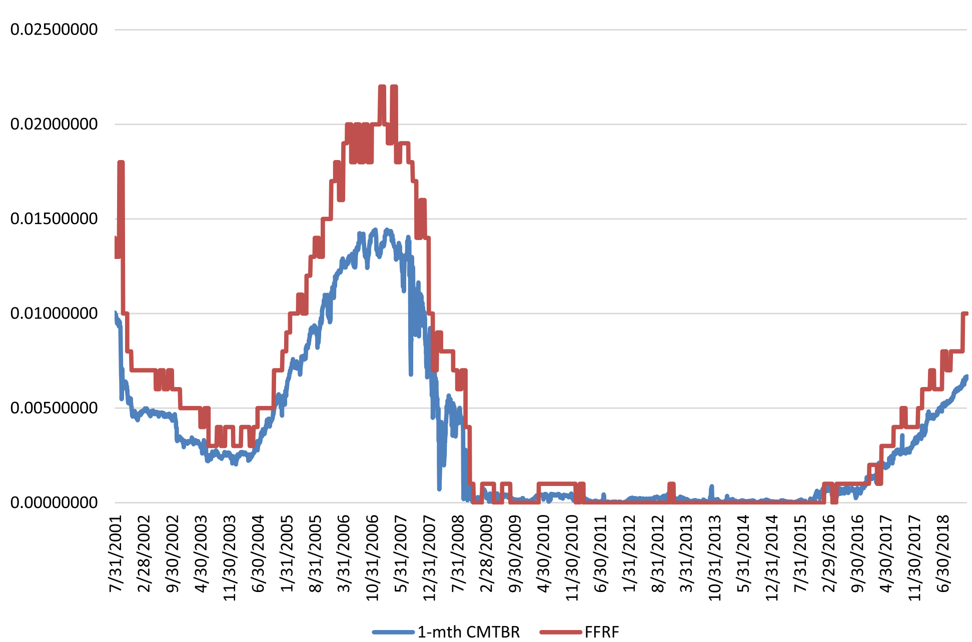

Figure 4 Daily risk-free rates comparison.

The following figure plots the one-month constant maturity yield (dark gray) versus the Fama–French risk-free rate (light gray) from July 31, 2001 to December 31, 2018. The figure shows that the daily Fama–French risk-free rate tends to be higher than the one-month constant maturity yield.

Table 7 Daily Fama–French and Weekend-Adjusted One-Month Constant Maturity Rate.

	Fama–French Risk-Free Rate				One-Month Constant Maturity Rate			
	2004	2006	2014	2018	2004	2006	2014	2018
1	0.004	0.016		0.006	0.0080	0.0122		0.0040
2	0.004	0.016		0.006	0.0027	0.0122		0.0040
3	0.004	0.016	0.000		0.0027	0.0122	0.0003	
4	0.004		0.000		0.0027		0.0002	
5	0.004		0.000	0.006	0.0026		0.0002	0.0122
6		0.016	0.000	0.006		0.0365	0.0002	0.0042
7		0.016	0.000	0.006		0.0122	0.0002	0.0042
8	0.004	0.016		0.006	0.0078	0.0122		0.0042
9	0.004	0.016		0.006	0.0027	0.0122		0.0042
10	0.004	0.016	0.000		0.0027	0.0122	0.0004	
11	0.004		0.000		0.0026		0.0002	
12	0.004		0.000	0.006	0.0026		0.0001	0.0128
13		0.016	0.000	0.006		0.0366	0.0002	0.0044
14		0.016	0.000	0.006		0.0123	0.0001	0.0046
15	0.004	0.016		0.006	0.0077	0.0123		0.0046
16	0.004	0.016		0.006	0.0027	0.0123		0.0046
17	0.004	0.016	0.000		0.0026	0.0123	0.0004	
18	0.004		0.000		0.0025		0.0002	
19	0.004		0.000	0.006	0.0025		0.0002	0.0137
20		0.016	0.000	0.006		0.0375	0.0002	0.0047
21		0.016	0.000	0.006		0.0128	0.0001	0.0046
22	0.004	0.016		0.006	0.0077	0.0128		0.0045
23	0.004	0.016		0.006	0.0026	0.0128		0.0045
24	0.004	0.016	0.000		0.0026	0.0128	0.0004	
25	0.004		0.000		0.0026		0.0001	
26	0.004		0.000	0.006	0.0026		0.0001	0.0138
27		0.016	0.000	0.006		0.0383	0.0001	0.0045
28		0.016	0.000	0.006		0.0129	0.0001	0.0044
29	0.004	0.016		0.006	0.0078	0.0128		0.0044
30	0.004	0.016			0.0027	0.0128		
31	0.004	0.016	0.000		0.0026	0.0127	0.0002	
Total	0.092	0.368	0.000	0.126	0.086	0.386	0.004	0.127

This table is a replication to Table 4 with daily one-month constant maturity rates adjusted for weekends. Notice that the Monday value is much more significant in the daily one-month constant maturity rate columns. The risk-free rates in this table are in percentages.

The last line of Table 7 provides the totals of the daily returns across the month and shows that FFRF and one-month CMTBR provide similar totals across the months reported. This finding suggests that either is a reasonable proxy for one-month risk-free security. The issue is that constant daily returns under FFRF does not reflect market data and could distort short-horizon and single-event studies.

To address the issue of short-horizon and single-event studies, we revisit the FOMC meeting event study of Cieslak *et al.* (working paper, 2016). Previously, we re-examined the FOMC meetings in 1995. We re-examine the FOMC meetings for 2006, which illustrates the most significant differences between the FFRF and the one-month CMTBR that is not during the financial crisis. In the interest of brevity, we do not create an additional table and instead just discuss the results here. We calculate results for week 2 (days 9–13) following the FOMC meeting dates. The average difference between the daily rates (one-month CMTBR minus FFRF) is 0.08 basis points per week. This value annualizes to 4.16 basis points. However, the average difference on Mondays during these weeks is 1.49 basis points, which annualizes to 77.48 basis points.

The differences here represent an arbitrage opportunity in the risk-free security. McGowan and Nosal (2020) examine foreign banks borrowing fed funds from the Federal Home Loan Bank (FHLB) and then holding the borrowed funds in their reserve accounts to earn the interest rate on excess reserves (IOER). The authors note that the IOER arbitrage becomes less attractive when the spread is 5 basis points or less. Our differences in rates over week 2 following FOMC meetings in 2006 are border-line economically significant and that the differences on Mondays during these weeks are economically significant.

6 Conclusion

Our analysis provides three essential results. First, the FFRF is constructed using several simplifying assumptions that provide returns that are not a good proxy of market data. Second, the choice of the FFRF as the risk-free rate does not alter the results relative to an improved proxy for the risk-free rate in long-horizon asset-pricing models and events studies. The lack of statistical differences results from the effect of long-run averaging. Lastly, we find an economic difference between FFRF and CMTBR in single events and a long/short analysis.

Following our results, we looked for an important paper that used FFRF in a short-horizon study where the implications of the results would change with a better proxy for the risk-free rate. We have not found an example. However, we did find something important for the profession: a difference in the use of T-bill data, which has been ongoing for decades.

Chen *et al.*'s (1986) study is the seminal paper on the empirical estimation of the APT (Arbitrage Pricing Theory). Chen *et al.* use monthly data from January 1953 through November 1983, a 371-month sample period.[25] The authors use Treasury rate data from Ibbotson and Sinquefield (1982). These data are the precursor to the Ibbotson database used in Fama and French (1992).

Published in the same year as Chen *et al.* (1986), Park and Reinganum (1986) document unusual price behavior in Treasury bills that mature at the turn of calendar months. They find that the last T-bill to mature in a month trades at a higher price (lower yield) than the adjacent bills. The authors find an average difference of 6.61 basis points per annum. Park and Reinganum (1986) use data from the CRSP Government Bond file. The CRSP data are from dealer quotes, which from 1962 through the end of the authors' sample came from the Federal Reserve Bank of New York (FRBNY) daily quote sheets.

Fama and French (1992) use the Ibbotson data. Musto (1997) examines year-end window dressing in commercial paper and uses T-bills as the benchmark rate. Musto compares December spreads with January spreads, so his analysis would be exposed to the month-end jump in the Ibbotson data. Musto (1997) uses Federal Reserve H15 data for three- and six-month T-bill rates and Bloomberg data for one-month T-bill rates.

However, Musto (1997) is a prime example of where the use of FFRF daily data is likely to cause problems. In the third table of the paper (p. 1574), the author compares December spreads to January spreads, and he reports the changes for the first day in January. The variable of interest for our study is the CP30—TB1M spread (30-day commercial paper minus the one-month T-bill). He finds that spreads decline 47.83 basis points (using annualized rates) from the last trading day of December through the first trading day of January. To highlight the importance of using market data, we collect the 30-day commercial paper rates for the H15 report and replicate Musto's analysis using both the FFRF and our suggested CMTBR. We find that the CP30—CMTBR spreads declines by an average of 34 basis points, which is consistent with Musto's results. After annualizing FFRF, we find the CP30—FFRF spread increases by an average of 11 basis points. Both results are economically significant in the money markets. However, using the FFRF data series would change Musto's conclusions.[26]

Cieslak *et al.* (working paper, 2016, JF 2018) use the Fama/French risk-free rate, which comes from the Ibbotson data. Bessembinder (2018) examines stocks in many buy and hold strategies relative to one-month T-bills over a sample period from 1926 through 2016. Bessembinder collects the one-month T-bill returns from Fama and French databases.

Gorton and Metrick (2012) examine repo runs during the most recent recession. They estimate the Libor-OIS spread as a proxy for fears of bank insolvency. They find spreads of 7.97 basis points (in annual rates) in the first half of 2007 (see, Gorton and Metrick, 2012, Table 2, Panel A), which increases to 58.71 basis points in the second half of 2007 and 108.10 basis points across 2018. The Libor-OIS spread is a measure of bank default risk, which is typically measured as a spread over T-bill yields. However, during the financial crisis, T-bill yields were subject to issues from flight-to-quality.

The point we are trying to make with this brief literature review is that over the last three decades, different branches of finance have been using different benchmarks for the risk-free rate. We document a difference in the use of T-bill data. The asset-pricing literature and long-run event studies use the FFRF data, while money market papers do not. In all of the money market papers of which we are aware, the FFRF is never the choice as a data source for T-bill rates. In other words, in studies where the issues with FFRF could matter, FFRF is not used. In studies where issues in FFRF are unlikely to change the interpretation of the results, FFRF is used. Researchers in the money markets are making informed choices. We believe that researchers in the long-run studies are also making informed choices, but the absence of any discussion on the issues raised here means that we cannot be sure.

Appendix A – Describing the Four-Week T-Bill

Cieslak *et al.* (2016) examine the pattern in U.S. excess stock returns from 1994 through 2015 using a "30-day" T-bill return as their reference rate when calculating excess returns. There are several essential points to be made about "30-day" T-bills relative to the authors' sample period.

First, the Treasury did not issue four-week bills throughout this sample period. The first four-week T-bill was issued in July of 2001. Accordingly, if researchers want a "one-month" instrument across this sample period, then care needs to be taken to not switch securities in 2001 because newly issued four-week bills are on-the-run while all other Treasuries will be off-the-run.

Second, the shortest-term T-bill that is available across this sample is a three-month bill, which is correctly described as a 13-week bill. This bill is issued typically with 91 days to maturity. As the bill matures, it will have 30 days to maturity at one point in time. However, the authors claim to use a 30-day reference rate across a five-day window, which is not possible because the Treasury issues new bills only once per week.

Finally, a 13-week T-bill with approximately 30 days to maturity is off-the-run. Barclay *et al.* (2006) describe on-the-run Treasuries as newly issued with very active trading. When a new Treasury security is issued, the previous issue goes off-the-run, and its trading can decline by more than 90%. While Barclay *et al.* (2006) focus on Treasury notes, Moore and Winters (2014) revisit the issue for T-bills and also find that T-bill volume falls and spreads increase as bills go off-the-run. Griffiths *et al.* (2010) show that dealers include an illiquidity premium in the quoted spreads for off-the-run T-bills. Their results suggest an illiquidity premium varying from 2 to 8 basis points. Accordingly, the use of an off-the-run 30-day T-bill means that the reference rate is likely to include a premium for illiquidity. Whether or not to include an illiquidity premium in the reference rate when calculating excess stock returns depends on the sample of stocks. However, the illiquidity premium is something that researchers should be aware of when choosing their reference rate.

Thus, the description of the 30-day T-bill reference rate raises several concerns and demonstrates that the authors cannot do what they initially set out to do in their study.

Appendix B – Additional Details on Ibbotson Data Sources for the One-Month T-Bill Return

Ibbotson uses the CRSP U.S. Government Bond File from 1926 to 1976. The discussion of the CRSP US Treasury data sources follows:

"Prior to January 1962, treasury data were obtained from a number of different sources. These sources include the *Wall Street Journal*, Salomon Brothers, Inc., and the Bank and Quotation Record.

Beginning with January of 1962, the majority of prices came from the Composite Closing Quotations for US Government Securities compiled by the Federal Reserve Bank of New York (FRBNY). The time at which the quotes were compiled was related to the fedwire deadline the FRBNY set for the transfer of securities. The deadline was set for 2:30 p.m. Eastern Time, but was regularly extended as much as three-quarters of an hour. The FRBNY trading desk began a "closing run" at 3:00 p.m. The reference to "closing quotations" from 1962 to 1984 probably refers to the "closing run" at the FRBNY. With the close of the day on October 15th, 1996 the FRBNY discontinued publication of composite quotations.[27]"

Duffie (1996), in the Appendix to his paper, discusses details and issues with the Fed quote sheets. Ibbotson uses *Wall Street Journal* prices for 1977-present. Griffiths *et al.* (2010) use the *Wall Street Journal* data and provide the following description of their data, which provides the relevant details for the Ibbotson data.

"The daily data for off-the-run T-bills (from 8/1/89 through 10/15/96) are collected from Treasury security quote sheets compiled by the Federal Reserve Bank of New York. The Fed quote sheet data are collected from market participants (Treasury securities dealers) that the Fed believes to be reliable. The Fed produced its daily quote sheet prior to 8/1/89, but we do not have access to those sheets. Instead, from 6/13/88 through 7/31/89, we collect data from the Wall Street Journal. The Wall Street Journal reporting format is identical to the Fed quote sheets for the period from 6/13/88 through 7/31/89. Beginning on 8/1/89, we have access to both the Fed daily quote sheet and the Wall Street Journal, and random comparisons between the two sources find them to be identical on each day examined. Data for the period from 10/16/96 through 6/30/01 are also collected from the Wall Street Journal. For the period from 10/16/96 through 6/30/01, the Federal Reserve did not produce its daily quote sheet,[28] so the Wall Street Journal had to find an alternate source for its Treasury securities data; the data reported during this period are provided by Cantor Fitzgerald."

This description shows that the Ibbotson continues to collect data from the Fed quote sheets through October 15, 1996 when the Fed discontinued the sheets. From that point forward, the *Wall Street Journal* retained its reporting format but collected the data from Cantor Fitzgerald.

These additional details indicate that Ibbotson retains a process that uses dealer quotes to provide its one-month T-bill return. CRSP also continues to use dealer quotes but has a different source for its quotes. Thus, all the data used to calculate T-bill returns in Fama–French use dealer quotes instead of market prices.

Endnotes

1 The quote that we use to motivate our analysis first appeared in a 2016 working paper, when we started this research project, and it remains unchanged in the final published article.

2 We acknowledge that number of days to maturity is frequently used to describe T-bills.

3 When we started this project in 2018, Fama and French (1992) had over 19,000 Google Scholar cites.

4 The Fama–French data are available at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

5 We note that the FRED database at the Federal Reserve Bank of St Louis uses months to describe T-bills.

6 Currently, the Treasury auctions a new 4-week bill every week. The bill is issued on Thursday and matures on the fourth Thursday following issue. This issuance pattern is the standardized. The actual issue day, maturity day and number of days to maturity are subject to holidays. From its inception through the 12/6/18 issue, the standard 4-week bill was issued on Thursday. The issue following 12/6/18 was on 12/11/18, which is a Tuesday. From the 12/11/18 issue forward the issue day of the week has floated. The tentative Treasury auction calendar in May 2019 shows the 4-week bill settling in for a Tuesday issue through the end of 2019. See Appendix A for further details.

7 See Appendix B for further details.

8 Daily one-month CMTBR rates data are available from the https://fred.stlouisfed.org/series/DGS1MO.

9 Op. cit.

10 In 2006, Morningstar acquired Ibbotson Associates Inc.

11 Morningstar's definition for the one-month risk free return is: "The Ibbotson U.S. 30-day Treasury Bill Index is compiled from *Wall Street* Journal prices for 1977-Present and the CRSP U.S. Government Bond File from 1926 to 1976. Each month a one-bill portfolio containing the shortest-term bill having not less than one month to maturity is constructed. To measure holding period returns for the one-bill portfolio, the bill is priced as of the last trading day of the previous month-end and as of the last trading day of the current month." The Fama (1975) method is not exactly the same as the stated definition. The analyst we spoke to suggested that the Fama method is used because T-bills with only a few days or less to maturity are not actively traded. GovPX data for 1997 (generally believed to be most representative year) show an average of about six sales of T-bill on the last trading day of the month for bills that are about to maturity. Moreover, the average lot size for end-of-month trades on T-bills about to mature in 1997 is $21 million. The median lot size is $10 million. The average lot size for end-of-month trades on T-bills closest to 30 days to mature but with 30 days or more to maturity in 1997 is $20 million. The median lot size is $10 million.

12 See Appendix B for additional information regarding the data and construction.

13 T-bills are issued to mature on Thursday to align with the auction calendar. Adjustments are made for Thursday holidays. T-bills that would mature on Thanksgiving are adjusted to mature on Friday. T-bills investors that are bidding in an auction can pay for the new T-bill by rolling a maturing bill on settlement day. On settlement day the Fed pays the buyer the difference between the value of the maturing bill and the price of the new bill.

[14] The first appearance of the risk-free return data appears in Fama and French (1992).

[15] Recall that due to quantitative easing, T-bill rates were at a historical low and essentially zero during 2014.

[16] Connolly (1989) examines day-of-the-week and weekends effects using daily returns on the S&P 500 index, the CRSP equally-weighted index and the CRSP value-weighted index. His results suggest that both effects disappear after 1975. Griffiths and Winters (1995) examine the Federal Reserve bank settlement process and show the impact of market closed weekends on the process.

[17] We discuss the use of the 4-week T-bill in Section 5 and suggest the best available proxy for a one-month risk-free rate going forward.

[18] FRED (Federal Reserve Economic Data) has a variety of daily series for T-bills. The secondary market daily series for bills begin: (1) 1/4/54 for three-month bills, (2) 12/9/58 for six-month bills, and (3) 7/15/59 for one-year bills. Accordingly, all three series start early enough in time to work for the risk-free rate with the Fama–French start date of July 1963. Since all three work, the best choice is the shortest maturity: the three-month bill. There are a variety of issues with using secondary market data on the three-month bill as the risk-free rate. First, the daily change in the secondary market yield contains two components: (1) market rate changes and (2) mechanical price appreciation. Second, the time series uses the on-the-run bill, which changes with each new auction. This means that with every auction the time series breaks and moves from a bill with less than 90 days to maturity back to a bill with generally more than 90 days to maturity. These issues can be addressed, but the cost to the researcher of fixing the secondary market time series likely outweighs the benefit.

[19] Constant maturity rates are derived by the Treasury Department using a quasi-cubic hermite spline function. The inputs are daily indicative, bid-side market quotations for on-the-run securities. The current inputs are the most recently auctioned 4-, 8-, 13-, 26-, and 52-week bills; the most recently auctioned 2-, 3-, 5-, 7-, and 10-year notes; the most recently auctioned 30-year bond; and composite rates between the 10- and 30-year maturity range.

[20] We ran the sub-periods reported in Fama and French (1992) and found qualitative similar results. For the sake of brevity, we only report the full sample results in the manuscript. The complete replication is available upon request from the authors.

[21] Specifically, we calculate the spread between excess returns generated from the weekend adjusted daily CMTBR and the daily FFRF. The average spread for week 2 is 0.0102%. For week 4, we find an average spread equal to 0.0070%.

[22] That is, we use the EWRETD collected from CRSP on WRDS and 2 weeks after 1995 FOMC meetings (days 9–13).

[23] To convert the daily FFRF to a monthly rate, we multiply by the exact number of trading days in that given month. This approach is consistent with how the rate is constructed. We do not use weekend adjusted CMTBR in this analysis because the rate is converted to monthly data.

[24] The four-week T-bill is typically issued with 28 days to maturity. Historically, the four-week bill was issued on Thursdays. This means that secondary market data on Fridays is for a bill with 27 days to maturity. Mondays have 24 days. Tuesdays have 23 days. Wednesdays have 22 days. Then Thursdays reset to 28 days with the issue of the next new bill.

[25] Recall our earlier discussion in Section 2 on the non-existence of one-month T-bills during this time period. As such, we cannot replicate their results using our suggested risk-free rate.

[26] We note that similar analyses on money market rates across the year-end by Griffiths and Winters (1997, 2005)' find that spreads decline in the new year.

[27] "The start of the day, October 16, 1996, our source for daily and monthly price quotations, maturity dates, and coupon rates changed to GovPX, Inc. GovPX receives its data from five inter-dealer bond brokers. Live, intra-day bids, offers, and transactions in the active over-the-counter markets among these primary dealers are the source of GovPX's 5 p.m. End-of-day US Treasury prices. GovPX was acquired by ICAP in 2008. Beginning in February 2009, CRSP released its daily and monthly treasury databases using the new ICAP data."

[28] When the Fed stopped providing its daily quote sheet it made the following statement: "The Federal Reserve Bank of New York has ended publication of the daily treasury securities price quotes. The final issue was posted to the ftp site on October 15, 1996. This reflects the Fed's decision to discontinue daily collection and compilation of Treasury prices from its trading counterparties. No other Federal Reserve statistical releases

will be affected. There are various sources that provide similar information on the Internet.

References

Barclay, M., Hendershott, T., and Kotz, K. (2006). "Automation vs. Intermediation: Evidence from Treasuries Going off the Run," *Journal of Finance* **61**, 2395–2414.

Bessembinder, H. (2018). "Do Stocks Outperform Treasury Bills?" *Journal of Financial Economics* **129**, 440–457.

Chen, NF., Roll, R., and Ross, S. A. (1986). "Economic Forces and the Stock Market," *The Journal of Business* **59**(3), 383–403.

Cieslak, A., Morse, A., and Vissing-Jorgensen, A. (2016). "Stock Returns over the FOMC Cycle," working paper.

Connolly, R. (1989). "An Examination of the Robustness of the Weekend Effect," *Journal of Financial and Quantitative Analysis* **24**, 133–169.

Duffie, D. (1996). "Special Repo Rates," *Journal of Finance* **51**, 493–526.

Fama, E. (1975). "Short-Term Interest Rates as Predictors of Inflation," *American Economic Review* **65**, 269–282.

Fama, E. and French, K. (1992). "The Cross-section of Expected Stock Returns," *Journal of Finance* **47**, 427–465.

Gorton, G. and Metrick, A. (2012). "Securitized Banking and the Run On Repo," *Journal of Financial Economics* **104**, 425–451.

Griffiths, M., Lindley, J., and Winters, D. (2010). "Marketing-Making Costs in Treasury Bills: A Benchmark for the Cost of Liquidity," *Journal of Banking and Finance* **34**, 2146–2157.

Griffiths, M. and Winters, D. (1995). "Day-of-the-Week Effects in Federal Funds Rates: Further Empirical Findings," *Journal of Banking and Finance* **19**, 1265–1284.

Griffiths, M. and Winters, D. (1997). "On a Preferred Habitat for Liquidity at the Turn-of-the-Year: Evidence from the Term-Repo Market," *Journal of Financial Services Research* **12**, 21–38.

Griffiths, M. and Winters, D. (2005). "The Turn of the Year in Money Markets: Tests of the Risk-Shifting Window Dressing and Preferred Habitat Hypotheses," *Journal of Business* **78**, 1337–1364.

Ibbotson, R. and Sinquefield, R. (1982). "Stocks, Bonds, Bills and Inflation: The Past and Future," Published by Financial Analysts Research Foundation.

McGowan, J. and Nosal, E. (2020). "How Did the Fed Funds Market Change When Excess Reserves Were Abundant?," FRBNY *Economic Policy Review* **26**, 1–15.

Moore, M. and Winters, D. (2014). "When is a Treasury Security on-the-Run?," *The Financial Review* **49**, 77–88.

Musto, D. (1997). "Portfolio Disclosures and Year-end Price Shifts," *Journal of Finance* **52**, 1563–1588.

Park, S. Y. and Reinganum, M. R. (1986). "The Puzzling Behavior of Treasury Bills that Mature at the Turn of Calendar Months," *Journal of Financial Economics* **16**, 267–283.

Sharpe, W. (1964). "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk," *Journal of Finance* **19**, 425–442.

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