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HORIZON-ADJUSTED PORTFOLIO PERFORMANCE MEASURE

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This paper presents a portfolio performance measure that accounts for the investment horizon assuming both risk and loss aversion as suggested by Tversky and Kahneman's CPT framework. The optimal portfolio risk premiums of such investors decrease with the length of the investment horizon and our simulations indicate that the decrease is drastic. The suggested measure is theoretically-based and provides a user-friendly metric for gauging the appropriate relationship between the horizon and the investor's optimal portfolio composition. Applying the methodology will likely lessen myopic behavior of investors and induce an increase of their portfolio's weight on equities for longer term investors.

Key Takeaways:

Simulations of multi-period returns that are based on monthly returns of S&P-500 index data over a period of 10 years starting in 2008 show:

- Investors' optimal premiums to compensate for risk and potential loss drop by as much as 86% and up to 93% as their investment horizon extends from 1 to 60 months. The effect of the horizon on the premium depends on the degrees of the risk aversion and loss aversion. In the period covered by our analysis, the certainty equivalent performance of the investors' risky portfolio is above the risk-free rate only when the holding period is longer than 4–8 months.
- Common performance ratios lack inherent adjustment for the investment horizon, as they are typically calculated by employing short-term data. The suggested methodology overcomes this limitation and may encourage a departure from a myopic behavior.
- Our methodology provides investors and portfolio managers with a quantitative metric for gauging the tradeoff between the optimal portfolio composition and the horizon length, given some basic characteristics of the ultimate investor attitude toward risk and loss.

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1 Introduction

There is a long lasting debate among academics and practitioners regarding the effect of investors' horizon on their portfolios' optimal equity level.[1] Samuelson (1969), Merton and Samuelson (1974) and Bodie (1995) showed that for investors with constant relative risk aversion (CRRA) utility, the investment horizon does not affect the optimal level of the portfolio's equity. Mossin (1968) noted that in the case of constant absolute risk aversion utility function (CARA), the horizon effect is weak.

On the other hand, the seminal work of Kahneman and Tversky's (1979) Prospect Theory, Tversky and Kahneman's (hereafter TK; 1992) Cumulative Prospect Theory (CPT) and Mehra and Prescott's (1985) Equity Premium Puzzle (EPP) imply that longer horizons justify higher risk taking. Benartzi and Thaler (BT; 1995) claim that short-term performance reports cause loss-averse investors with long investment horizons to make myopic investment decisions and act as if their average horizon is only one year, hence they fail to utilize the benefits of time diversification.

Indeed, it is well recognized, theoretically and empirically, that performance measures must reflect differences in the investors' horizons. For example, Sharpe (1994) showed that when the returns distribution is iid across time, the Sharpe ratio for a horizon of length t increases with time as a function of the square root of t. Francois and Hubner (2020) also use this relationship to compute yearly Sharpe ratios using monthly data. Levy (2017) notes that the Sharpe ratio is sensitive to the investor's planned horizon and needs to be corrected prior to ranking the performance of portfolios if the data are based on time intervals that do not match the planned horizon, and no such adjustment is required for the geometric mean return where the returns distribution is iid across time.

Additionally, it is to be expected that portfolio performance be assessed differently by investors that have diverse attitude toward risk and loss. Zakamouline (2011) recognized this fact and developed, under some concrete assumptions about the form of the investor's utility function, a closed-form solution for the investor's portfolio performance measure. Farinelli *et al.* (2009) also recognize the fact that optimal asset allocation must fit investors' goals and thus must lead to parameter-dependent performance ratios. They find satisfactorily good match between asset allocation and corresponding risk profile. Similarly, Pezier (2011) employs a maximum certainty equivalent excess return criterion, or equivalent criteria, adapted to the investment circumstances. Levy (2017) compares the Sharpe ratio, Jensen's Alpha and the geometric mean as performance measures. He demonstrates the bias of the Sharpe ratio when using short-term data to evaluate the performance of long-term horizon portfolios, a bias that is not shared by the geometric mean. Indeed, this drawback is shared by all other commonly used performance measures. Using an experimental test Levy and Levy (2017) found that assuming PT preferences, the optimal asset allocation shifts drastically toward 100% equity if the investment horizon is as short as one, two, or three years. These findings are similar to ours but are not in line with observed investor behavior. As we argue, this can be easily explained by the fact that the horizon length is generally not deterministic and is often defined as "x number of periods ahead".

Practitioners typically assume positive relationship between the investment horizon and the desired portfolio risk level, a practice that is well reflected in the "100 minus age" equity proportion rule of thumb used in pension accounts and in the well-known Chilean pension programs.

The main purpose of this paper is to demonstrate the opportunity open to investors with diverse horizons and alternative attitudes toward risk and loss, to enhance expected wealth by using optimal risk premiums that suite their preferred horizon length. Realizing the dramatic reduction of the risk premium as the investment horizon lengthens, will likely mitigate the myopic investors' behavior who tend to focus on short-term returns and fail to sufficiently account for the time diversification of returns.

The paper is organized as follows. Section 2 presents the methodology for deriving the horizon risk premiums (HRPs) expressed in short periods terms. Section 3 presents empirical data to estimate the HRPs in a bootstrapping simulation setting TK's value function. Section 4 examines the impact of the risk aversion and loss aversion parameters on the optimal selected risk and the HRPs. Section 5 presents the "efficient frontier" between achievable short-term values and investment horizons. Section 6 presents a summary and offers some conclusions.

2 Methodology

Our analysis is based on a two-asset portfolio that consists of a proportion α invested in a risky asset (i.e., the equity component) and a proportion $(1-\alpha)$ invested in a risk-free asset. The equity component yields a random return $\tilde{r}(e;t)$ in each short period t, while the risk-free asset yields a known return r_f. The equity return is assumed to be identically independently distributed (iid) across the short periods. The portfolio is rebalanced to its original proportions at the beginning of each short period, t. Hence, the portfolio's multi-period rate of return for a horizon of T periods, $\tilde{r}(T)$, is given as follows:

$$\tilde{r}(T) = \prod_{t=1}^{T}[1 + (\alpha\,\tilde{r}(e;t) + (1-\alpha)r_f)] - 1 \qquad (1)$$

The expected one-period rate of return on the risky portfolio derived from the expected cumulative rate for horizon of length T is given as follows:

$$\bar{r}(1;T) = [1 + E(1+\tilde{r}(T))]^{1/T} - 1 \qquad (2)$$

For each horizon T, the portfolio's expected utility is expressed as follows:

$$E[U(\tilde{r}(T)] \qquad (3)$$

The certainty equivalent rate for that horizon given a specific utility function is noted as follows:

$$CE(T) = U^{-1}(E[U(\tilde{r}(T))]) \qquad (4)$$

Given the CE rate for the T holding period, we derive the one-period certainty equivalent value:

$$CE(1;T) = \{1 + E[CE(T)]\}^{1/T} - 1 \qquad (5)$$

The portfolio's one-period horizon risk premium, $HRP(1;T)$, is simply the spread between the portfolio's one-period return and the one-period certainty equivalent rate as derived from the T holding period horizon:

$$HRP(1;T) = \bar{r}(1;T) - CE(1;T) \qquad (6)$$

Conversely, the portfolio's one-period certainty equivalent performance measure for any horizon T is:

$$CE(1;T) = \bar{r}(1;T) - HRP(1;T) \qquad (7)$$

The one-period certainty equivalent rate, $CE(1;T)$, is a single rate that provides the same utility as the expected utility of the relevant risky portfolio return for a given horizon.

If time diversification is beneficial, the one-period $HRP(1;T)$ is likely to decrease with the horizon, while the one-period certainty equivalent rate, $CE(1;T)$, is likely to increase with the horizon, inducing greater risk taking by long-term investors. We therefore use the rate $CE(1;T)$ as a yardstick for comparing the performance of alternative risky portfolios that have a horizon T.

The apparent major weakness of our performance measure is the fact that it is derivable from the investor's utility function; hence, it is not a single objective measure suitable for all investors. However, the attempt to use an objective performance measure such as Sharpe ratio that fits all can work well only when the "separation theorem" holds. However, an objective unique performance measure surely cannot fit investors with diverse horizons. Indeed, financial consultants commonly tailor portfolios composition to fit their clients preferred attitude toward risk and loss, willingness to use different classes of securities and the length of their investment horizon. Given such investors' data, the use of the certainty equivalent provides a user-friendly quantitative and theoretically based measure that exploits the potential benefit of time diversification.

3 Estimating the HRP and CE Using Empirical Data in a Bootstrapping Simulation Setting

In this section, we demonstrate how our method can be applied, by estimating HRP for the S&P-500 index[2] via bootstrapping simulations to determine the expected $HRP(1;T)$ for horizons that range from 1 to 60 months with different levels of investment in equity and risk-free assets.[3] Applying TK's value function to our multi-period returns, we have the following:

$$v(\tilde{r}(T)) = \begin{cases} \tilde{r}(T)^{\alpha} & \text{if } \tilde{r}(T) \geq 0 \\ -\lambda(-\tilde{r}(T))^{\beta} & \text{if } \tilde{r}(T) < 0 \end{cases} \quad (8)$$

The prospect utility (value) function of a risky portfolio P in a sample setting is:

$$V = \sum_{i=1}^{I} \pi_i v(r_i(T)) \quad (9)$$

where π_i represents the decision weight related to the i^{th} sample outcome ($i = 1, 2, \ldots, I$). In portfolio analysis, $r_i(T)$ is the i^{th} rate of return on the invested capital in the portfolio for a horizon of T short periods. It is reasonable to assume that π_i is simply the probability of obtaining the rate of return $r_i(T)$. Our "base case" for Equation (7) uses the parameters suggested by TK, namely, $\alpha = \beta = 0.88$ and $\lambda = 2.25$. The simulations used monthly rates of return of the S&P-500 index ("the equity component") and the monthly rates of return of the one-month TBs ("the risk-free component") over the 120 months during the period February 2008 to January 2018. The average monthly rate of return of the TBs during this 10-year period serves as our risk-free rate of return in the bootstrapping simulations procedure. The 120 monthly rates of return on the S&P-500 index serve as a pool, from which we simulated 2,000 random draws for each horizon length, producing a total of 120,000 (= 2,000 × 60) random draws. We use these random outcomes to compute the returns of portfolios that differ with respect to their equity levels for each horizon, namely, the proportion of the risky assets component vs. the risk-free asset component. Table 1 presents some of the data's basic statistics.

The sample annualized average TBs and S&P-500 index rates of return over this period were 1.28% and 8.66%, respectively. The TB rates are highly autocorrelated; they are positively skewed and more concentrated in the center relatively to a normal distribution (i.e., have negative kurtosis).[4] The opposite holds for the S&P-500 index rates of return: the autocorrelation is low, they are negatively skewed, and their kurtosis is positive.

Table 1 Basic statistics of the monthly rates of return: S&P-500 and one-month Treasury bills February 2008 to January 2018.

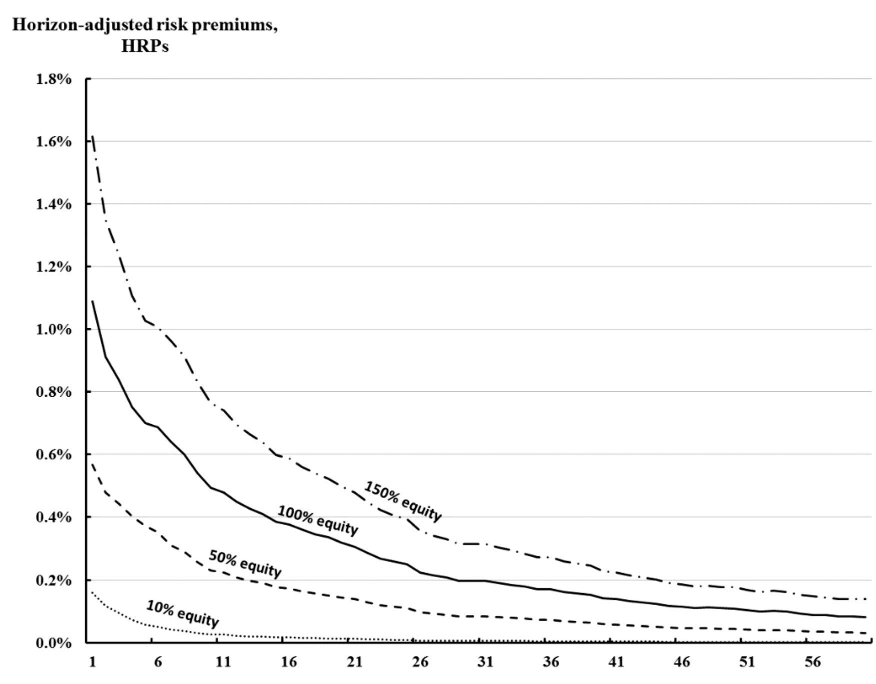
	TBs	S&P-500
Average monthly rate	0.1067%	0.6943%
Standard deviation	0.1547%	4.3155%
Skewness	110.556	−84.064
Kurtosis	−55.630	204.447
Correlation	0.0493	
Autocorrelation $(t, t-1)$	0.9954	0.1435
Significance	0.00	0.12

Aiming to obtain a clear view of the effect of the portfolio's equity levels on the one-period $HRP(1; T)$ and $CE(1; T)$, we present results of four portfolios with different equity levels of 10%, 50%, 100% and 150%, respectively, and they are rebalanced every month.

Exhibit 1 shows the monthly (i.e., one-period) horizon risk premiums, $HRP(1; T)$, under the CPT model applying our "base case". It is evident that under this model, the premiums are very high for short horizons but drop sharply with the horizon length. For example, for portfolio with 100% equity, the one-period premiums for horizons of 1, 12, 24, 36, 48 and 60 months are 1.09%, 0.45%, 0.26%, 0.17%, 0.11% and 0.08%, respectively. The simulations indicate that for the "base case" the risk premium for 60-month holding period is less than a 10th of the risk premium for one-month holding period.

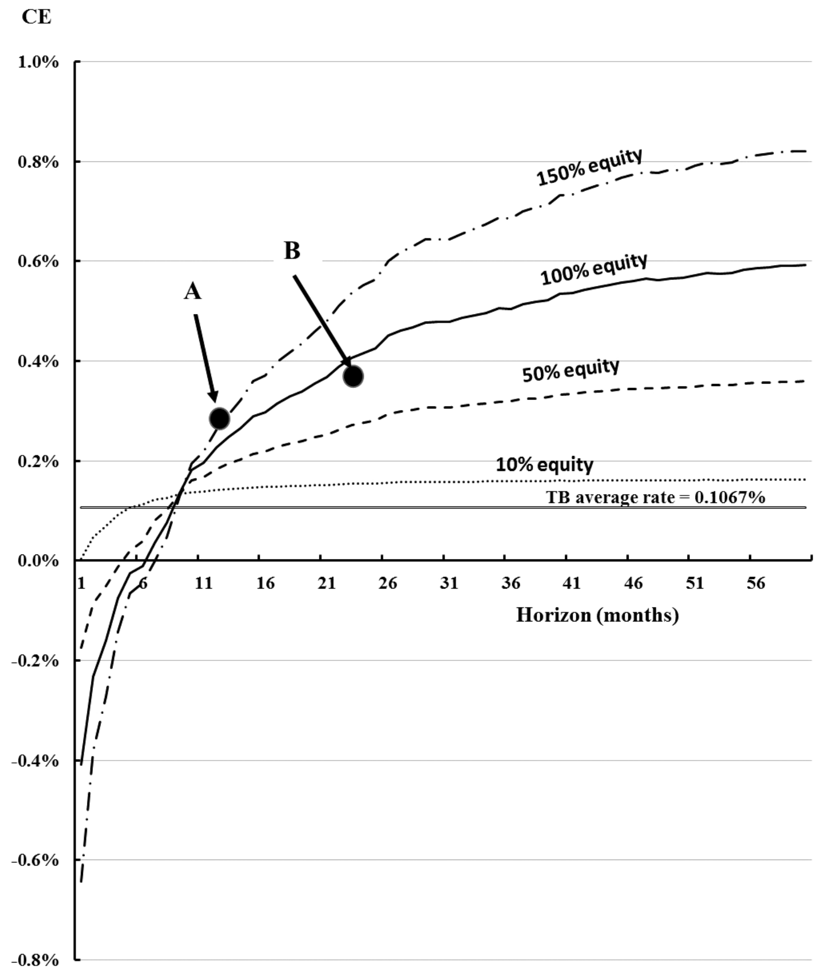
Recall that the monthly risk-free rate used in the simulations, 0.1067%, is the average TB rate in the 120 months from February 2008 to January 2018. In Exhibit 1, the line pertaining to portfolio with 150% equity is, in fact, a hypothetical line, as 150% equity implies margin purchase, which is

Exhibit 1 Monthly horizon risk premiums, $HRP(1; T)$ based on CPT value function with $\lambda = 2.25$ and $\alpha = \beta = 0.88$.



For each risk level, as measured by the proportion of equity vs. the risk-free asset in the portfolio, the graph presents the HRPs based on our bootstrapping simulation data.

Exhibit 2 Monthly average certainty equivalent returns, CE, for alternative portfolios and horizons based on CPT value function with $\lambda = 2.25$ and $\alpha = \beta = 0.88$.



For each risk level, as measured by the proportion of equity vs. the risk-free asset in the portfolio, the graph presents the CE rate based on our bootstrapping simulation data.

available only at a much higher margin-borrowing rate.

Exhibit 2 presents the one-period performance measure of CE values of our four portfolios for diverse horizons. The CE increases monotonically with the horizon for all equity levels. The rate of increase becomes greater as the equity level of the portfolio increases. For each level of equity, the marginal increase diminishes as the horizon becomes longer. Surprisingly, at about nine-month horizon, all the four simulated portfolios have approximately the same CE. The CE exceeds the risk-free rate only when the horizon is longer than 4–8 months. Referring to the horizon where the CE is equal to the risk-free rate as a "breakeven" horizon, we note that the higher the portfolio's equity proportion, the longer the "breakeven" horizon.

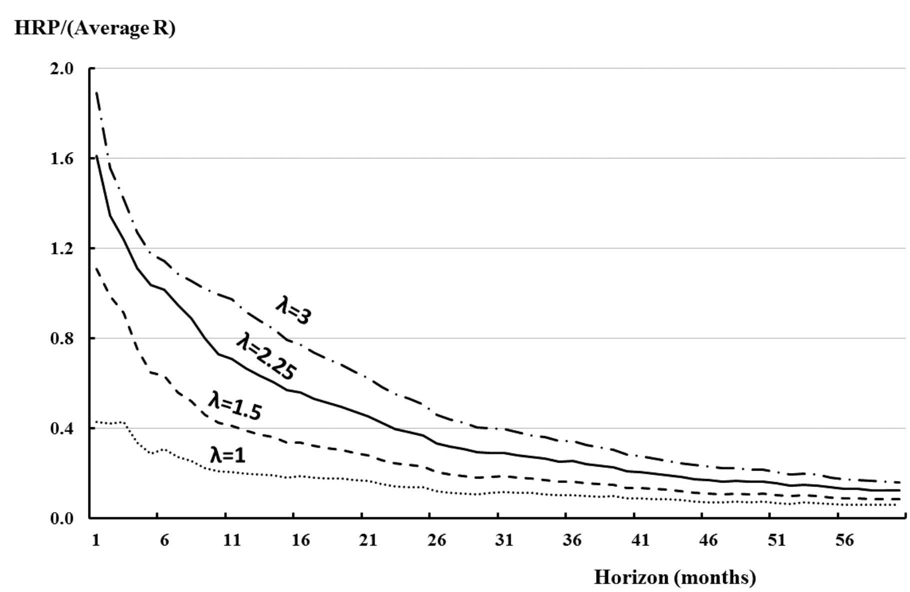
Exhibit 2 clearly demonstrates the need to consider the target horizon when ranking the performance of alternative portfolios. To observe this trend, assume that the graph of CE for 100% equity serves as the S&P-500 benchmark at different horizons for alternative equity managed portfolios. Portfolio A with a 12-month horizon has a CE of 0.3%, which is greater than the CE of the benchmark for that horizon. Portfolio B with a horizon of 24 months has a higher CE of 0.35%. However, this higher CE is below the benchmark's CE for investments with a 24-month horizon. It is evident that portfolio A outperformed portfolio B despite its lower CE.[5]

4 Changing the CPT Parameters

Since the certainty equivalent approach is utility dependent, we are able to examine the changes of the HRPs as the investor's attitudes toward risk aversion and loss aversion change. Exhibit 3 presents the ratio $\mathrm{HRP}/\bar{\mathrm{R}}$ for 100% equity portfolio using the CPT value function with loss aversion λ values of 3, 2.25, 1.5 and 1, while the risk aversion parameters α and β are set equal to 0.88 as suggested by TK. The effect of the loss aversion factor, λ, on $\mathrm{HRP}/\bar{\mathrm{R}}$ is particularly strong for short-horizon portfolios. For example, in the case of one-month horizon, the $\mathrm{HRP}/\bar{\mathrm{R}}$ is in the range of 1.89–0.43 for λ in the range of $\lambda = 1$ to $\lambda = 3$, respectively, while the $\mathrm{HRP}/\bar{\mathrm{R}}$ is in the range of 0.16–0.06 for the same λ levels, respectively, in the case of a 60-month horizon. It follows that the HRPs estimates of long-term investors merge into a relatively narrow range despite their marked difference in the degree of loss aversion. As noted above, in the absence of loss aversion ($\lambda = 1$) , the $\mathrm{HRP}/\bar{\mathrm{R}}$ is 0.43 for holding period of one month. It decreases gradually (almost monotonically) as the horizon becomes longer and reaches 0.06 at the 60-month horizon portfolio.

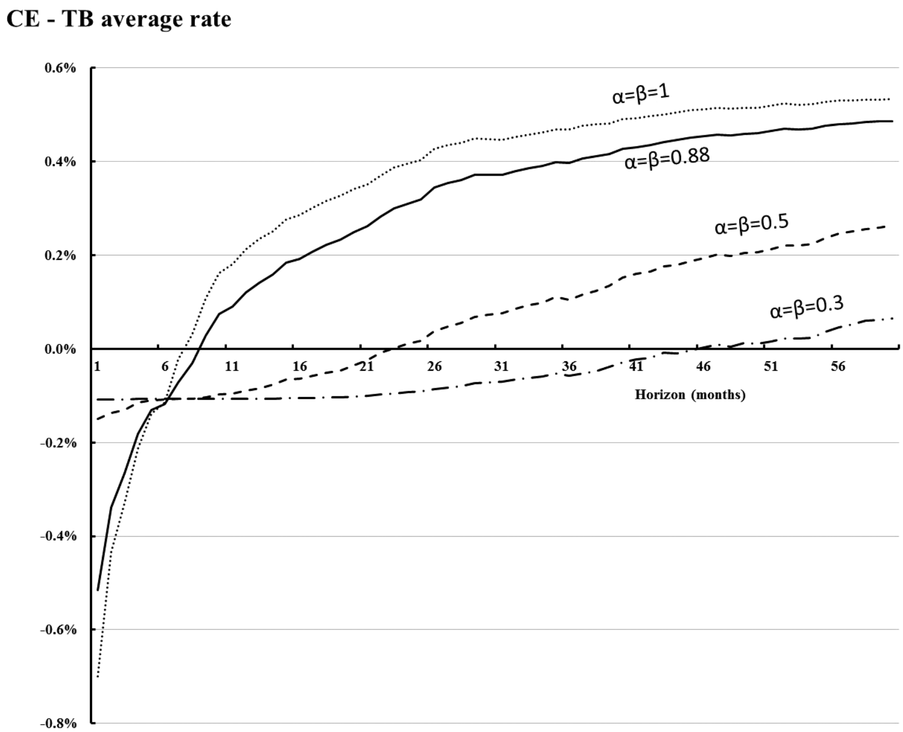
Exhibit 4 presents the way the average certainty equivalent excess return of the all-equity portfolio changes with the degree of risk aversion. When $\alpha = \beta = 1$, there is no risk aversion and the investor's attitude toward risk is motivated only by the loss aversion. In this case, when the horizon is only one-month long, there

Exhibit 3 Monthly HRPs relative to the S&P-500 average return, $\mathrm{HRP}/\bar{\mathrm{R}}$, based on CPT value function with $\alpha = \beta = 0.88$ and alternative λ values.



The graphs present the ratio $\mathrm{HRP}/\bar{\mathrm{R}}$ for alternative degrees of loss-aversion over a range of horizons based on our bootstrapping simulation data.

Exhibit 4 The effect of risk aversion on the CE return for the all-equity portfolio.



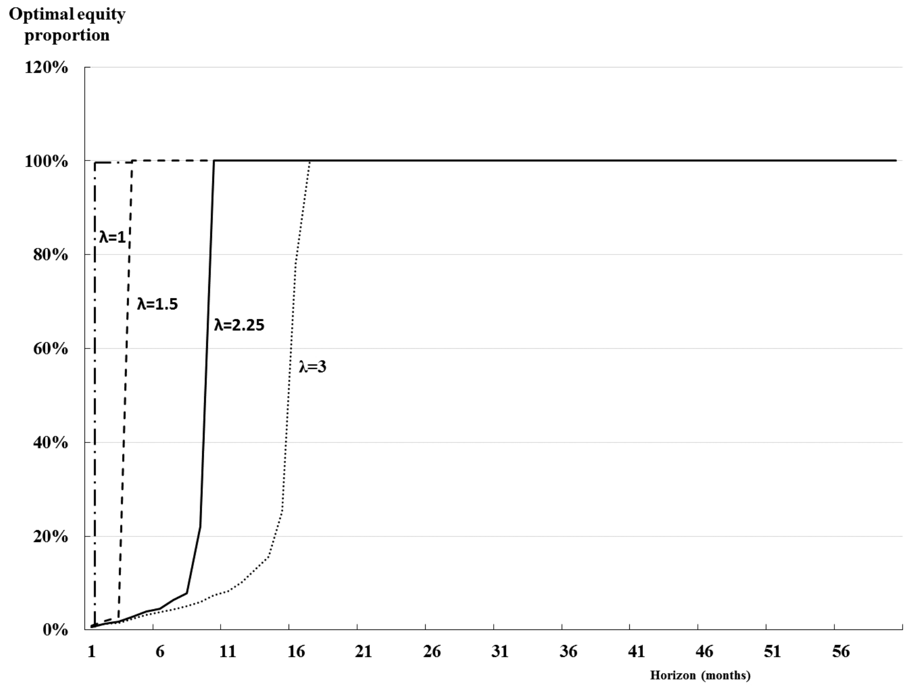
The monthly average certainty equivalent return in excess of the risk-free rate of return for varying degrees of risk aversion. The parameters used for the CPT value function are $\lambda = 2.25$ and $\alpha = \beta = 1$, 0.88, 0.5 and 0.3, and the graphs are based on our bootstrapping simulation data.

is a negative certainty equivalent premium of -0.70%. The premium turns positive only when the horizon is longer than seven months, and it reaches high level of 0.53% at the 60-month horizon. Lower α and β values represent higher-risk aversion level. Thus, for example, for $\alpha = \beta = 0.3$, the breakeven horizon for positive certainty equivalent excess return over the TB rate is 46 months. At 60 months, the excess return is only slightly greater than zero. Note that a difference in the level of risk aversion generates large differences in the expected excess certainty equivalent return of short-horizon as well as long-horizon portfolios.

Clearly, the optimal investment strategy is utility-dependent. Exhibit 5 presents the optimal portfolio risk level using our simulation data for alternative risk levels assuming the TK value function with risk aversion factors of $\alpha = \beta = 0.88$ and with the alternative loss aversion factors that we previously employed: $\lambda = 1, 1.5, 2.25$ and 3. As expected, the optimal equity level is very low for short-horizon investors, yet it increases dramatically toward 100% equity at a horizon that depends on the degree of loss aversion.[6] The higher the λ, the longer it takes before the dramatic shift toward 100% equity level occurs. As argued above, the reason why the equity level never exceeds 100% involves the margin requirements and the high borrowing rates. It is interesting to note that for loss neutral investors ($\lambda = 1$), the optimal portfolio consists of 100% equity even at the very short horizon of one month.

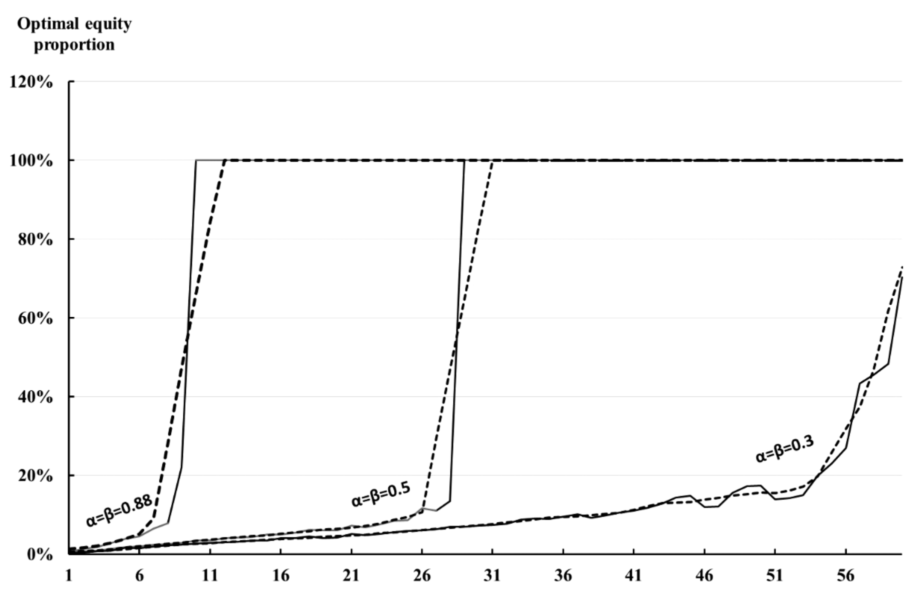
Exhibit 6 presents the portfolio's optimal proportion of the equity component from a different

Exhibit 5 Optimal equity levels for CPT value functions with alternative degrees of loss aversion.



The parameters used for the CPT value function are $\alpha = \beta = 0.88$ and $\lambda = 1, 1.5, 2.25$ and 3. The graphs are based on our simulation data.

Exhibit 6 Optimal equity levels for CPT value functions with alternative degrees of risk aversion.



The parameters used for the CPT value function are: $\lambda = 2.25$; $\alpha = \beta = 0.88$, 0.5 and 0.3. The solid lines represent the case of deterministic horizons, and the dashed lines assume an expected five-month moving average horizon. The graphs are based on our simulated data.

perspective. Here, we hold the loss aversion parameter at $\lambda = 2.25$, as noted in the "base case", and alter the risk aversion parameters α and β. The solid lines present the optimal portfolio composition assuming that the investment horizon is a deterministic future date that is well defined by the investor. Under this assumption, the optimal composition exhibits abrupt "jumps" of the optimal equity proportion toward the all-equity level at a horizon that depends on the risk aversion parameters (with a more gradual increase when $\alpha = \beta = 0.3$).

Clearly, a deterministic future horizon is not typical. In reality, the horizon is likely to be a planned gradual withdrawal of funds over several periods, such as in annuity plans (rather than a lump-sum withdrawal at one point of time) and/or a probabilistic horizon. In either case, the portfolio's optimal risk level will exhibit a gradual increase toward the 100% equity rather than a sudden "jump". In addition, the planned horizon may be a "moving" or "rolling" horizon rather than a fixed future calendar date (e.g., "one year ahead from today"), in which case the optimal equity level will be equal to the length of the "rolling" horizon and, barring exogenous new data, remain unchanged over time.

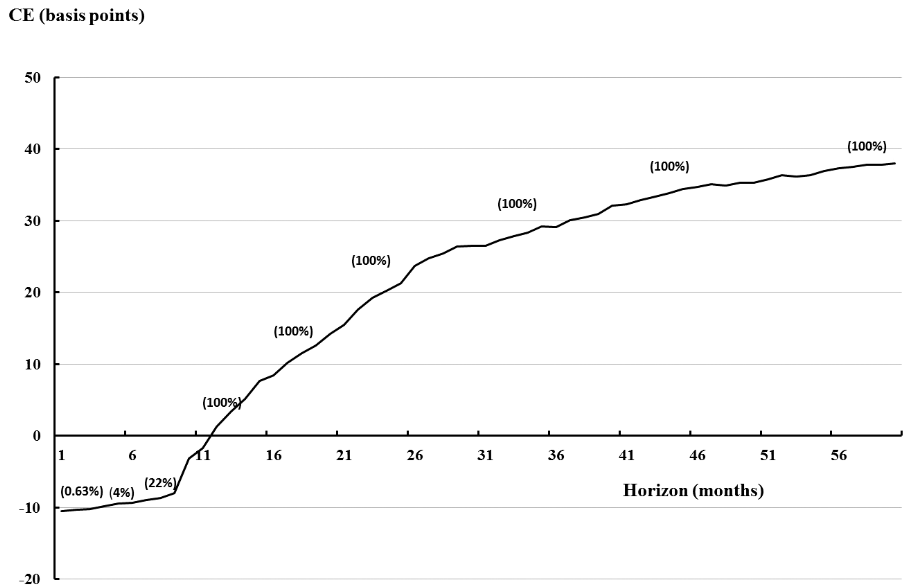
To illustrate the gradual change in the composition of the optimal portfolios, we present the dashed lines in Exhibit 6, which show the optimal equity proportions assuming that the expected horizon is a five-month moving average. When the risk aversion parameters are $\alpha = \beta = 0.88$, the optimal portfolio becomes all-equity if the horizon is 10 months or longer, provided the horizon is deterministic. However, under the moving average assumption, the climb toward all-equity level begins on the seventh month and reaches the all-equity level on the 12th month. An investor who is more risk averse, with $\alpha = \beta = 0.5$, finds it optimal to hold all-equity portfolio if the horizon is deterministic and equals 29 months or longer. With the moving average assumption, the shift to the all-equity portfolio begins on the 26th month and reaches the all-equity level on the 31st month. Finally, when $\alpha = \beta = 0.3$, the equity-only portfolio is optimal for horizons longer than 60 months.

5 The Efficiency Frontier with the CPT Utility

The optimal portfolio strategy for a given investor is chosen from the attainable tradeoff between the CE and the horizon. The graphs in the previous three exhibits indicate that the optimal equity level should be close to zero for very short-horizon investors, but it should increase quickly and monotonically toward 100% equity for investors with longer horizons. In a perfect market with borrowing rate equal to the lending rate, long-term horizon investors would even be better off with equity levels greater than 100%. Alas, the margin requirements (including the risk of receiving margin calls) and the high borrowing rates on margin purchases (ranging roughly from 5% to 8% per year, depending on the size of the loan among other factors) restrict the use of margin purchases both in the short and long terms.

Exhibit 7 presents our simulated "efficient frontier" for loss-averse investors with the "base case" parameters and deterministic horizon. For each horizon, the portfolio composition that maximizes the CE value for that horizon is presented. Taken as a whole, the graph displays the attainable highest CE value at each horizon. When the horizon is as short as one month, the equity proportion that maximizes the CE is almost negligible (0.63%), and the CE is negative and equals only -10.5 basis points per month. The maximum

Exhibit 7 The efficiency frontier showing the highest CE attainable level at each horizon choice along with the optimal portfolio risk level.



The graph is based on the CPT value function with $\lambda = 2.25$ and $\alpha = \beta = 0.88$. It presents the highest monthly CE value obtained using our bootstrapping simulation data. (The percentages in the parentheses indicate the portfolio's equity proportion that maximizes the CE for the given horizon)

CE remains negative for all horizons up to 11 months and becomes positive at a one-year horizon. Thereafter, it quickly increases into the positive domain, reaching approximately 40 basis points per month at a 60-month horizon. Perhaps more surprising is the way the optimal equity proportion changes with the horizon. At the five-month horizon, the equity proportion increases to 4% and subsequently exhibits a sharp increase, from 22% at a nine-month horizon to 100% as the horizon extends further to one year. At longer horizons, the optimal level of equity remains at 100%. As explained above, the switch to a high proportion of equity is likely to be more gradual in a real-life scenario since the horizon is unlikely to be a deterministic future date.

The quantitative interaction between the CE and the horizon may be used to improve investors' decisions with respect to the portfolio risk level, given their desired horizon.

6 Summary and Conclusions

Past theoretical studies indicate that longer investment horizons may not justify higher risk taking by risk averse investors. This argument induces myopic decision-making on the part of investors, with respect to their portfolio composition across time. However, more recently Tversky and Kahaneman (1992) argue that investors are motivated by both loss aversion and risk aversion. Consequently investors are better off taking on more risk (e.g., higher equity level) as their investment horizon becomes longer. Benartzi and Thaler (1995) claim that investors nevertheless tend to make myopic investment decisions since performance measures presented to them are based on short-term data. Indeed, and unfortunately, existing portfolio performance ratios fail to inherently reflect the potential benefit of time diversification. In practice, most portfolio managers indeed advocate higher risk taking for long-horizon investors, and our approach provides a theoretically based quantitative measurement that allows one to account for the horizon effect on the portfolio performance.

We introduce a user-friendly performance measure that is inherently sensitive to the investment horizon and can be easily gauged to investors' preferences, given the typical data investment managers poses regarding their clients' attitude toward risk and loss and about their planned horizon. Our empirical simulations using the S&P-500 index returns over a 10-year period indicate that longer horizons should significantly diminish investors' risk premiums. We found that for a wide range of loss-averse investors the appropriate (optimal) risk premium drops by as much as 86–93% as the investment horizon extends from 1 to 60 months. Our results indicate that an all-equity portfolio is optimal for most investors even at relatively short horizons. Given a deterministic horizon, the specific horizon at which the optimal equity level switches to an all-equity portfolio depends on the degrees of loss aversion and risk aversion, but it generally occurs abruptly around a "critical horizon" of only a few months. When a nondeterministic horizon length is assumed, the switch from a low equity to an all-equity optimal composition becomes gradual.

Endnotes

[1] A good review of the academic and practical debate on the effect of the horizon on equity investment is given in Van Eaton, R. D., and J. Conover, (2002).

[2] The CE of the S&P-500 index for a given horizon T, may be used as a benchmark for calculating the "Alfa" for portfolios with different planned horizons. Here, we merely demonstrate the use of our method using the S&P-500 index as a benchmark against which some portfolios could be ranked. Our intention is to apply the method to the actual ranking of alternative portfolios in a follow-up research.

[3] To simplify the graphical presentations of the results, we limit the presentation to only four levels of investment in equity: 10%, 50%, 100% and 150% of the total invested capital.

[4] These features are probably due, at least partially, to the high control and intervention of the Federal Reserve authorities.

[5] We show below that maximizing the CE for each given horizon by selecting an appropriate level of equity for that horizon generates an efficient frontier. Each investor can then select the optimal horizon by considering the available tradeoff between the horizon length and the certainty equivalent rate.

[6] Note, we assumed no margin purchases in the simulations, so that the optimal level of equity never exceeds 100%. The reason for imposing this constraint is the high borrowing rate in margin purchases and the unlikelihood of holding margin positions open in long horizon portfolios.

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