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FOOLED BY THE BLACK SWAN

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This paper offers a critical analysis of the normative theory of investment decisions as presented in Taleb's The Black Swan. I find that the relentless pursuit of positive black swans can lead investors to overprice opportunities, potentially triggering financial bubbles and crashes in the medium to long term. Conversely, underinvestment in the corporate bond and stock markets due to extreme aversion to negative black swans, can result in significant shortfalls in wealth creation for individuals and value destruction for corporations. In defense of the Nobel prize-winning works of Black and Scholes and Merton, I contend that the issue lies not in the scandal of prediction, but in the crafty manipulation of information, which has contributed to Finance becoming a more pseudo-discipline.

1 Introduction

"The Black Swan," authored by Taleb (2007, 2010), has indelibly shaped contemporary discourse on the evolution of socio-economic reality. This seminal work presents a dual theoretical framework that has polarized the intellectual community: a substantial majority lauds the metaphor of the black swan, while a discerning minority remains skeptical. The bifurcation in opinion arises from the interplay of two intertwined theories in this book: the *positive* theory, which elucidates the interaction between the fallible human mind and scalable probability distributions in socio-economic contexts, and the *normative* theory, which proposes methodologies for leveraging positive black swans while mitigating the adverse effects of negative black swans. The focus of this paper is the normative theory, which is crucial for black swan type investment funds. This paper provides a strong critique of the normative theory while acknowledging the substantive contributions of the positive theory.

The positive theory posits that our socio-economic reality predominantly resides in "Extremistan," a domain governed by black swans—rare, high-impact events that are unforeseen until their occurrence—and gray swans, which are rare, high-impact events following scalable probability distributions. In such an

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environment, traditional linear statistical methods are not only ineffective but also potentially perilous, particularly in the presence of cognitive biases such as the confirmation fallacy, the narrative fallacy, the problem of silent evidence, and the ludic fallacy. Conversely, the normative theory advocates for an investment strategy characterized by hyper-aggressive and hyper-conservative elements, exemplified by barbell portfolios that allocate substantial portions to extremely safe assets, such as U.S. Treasury Bills, while exposing a smaller fraction to highly speculative ventures, thus eschewing negative black swans.

This paper critiques the normative black swan theory, asserting that its portfolio strategy is myopic and prone to significant attrition in realistic option markets marked by high implied volatilities. Furthermore, the relentless pursuit of positive black swans can engender investor overpricing and subsequently precipitate market bubbles and crashes in the medium term. Additionally, the avoidance of negative black swans—investments in corporate stocks and bonds—can lead to significant deficits in wealth accumulation for individuals and value erosion for corporations, potentially triggering prolonged economic downturns.

The structure of this paper is as follows: Section 2 delineates the five principles of the normative black swan theory as articulated by Taleb (2007, Chapter 13). Section 3 addresses various fallacies associated with the normative black swan theory. Section 3.1 explores how high implied volatilities of deep out-of-the-money options, consistent with the volatility "smile," diminish the prospective upside of positive black swan strategies. Section 3.2 elucidates the propensity for such portfolio strategies to hemorrhage over time, thereby generating negative black swans in the intermediate to long term. Section 3.3 examines the paradoxical economic disequilibria that arise from the universal pursuit of positive black swans, culminating in financial bubbles and subsequent crashes, as exemplified by historical phenomena such as the tulip mania, the South Sea bubble, and the dot-com bubble. Section 3.4 draws parallels between positive black swans and lottery tickets, arguing that the latter, due to their higher scalability and historically superior upper limits on returns, may offer a more viable investment strategy. Section 3.5 discusses the macroeconomic implications of a collective aversion to "medium-risk" investments exposed to negative black swans, such as corporate loans, bonds, and stocks, potentially resulting in protracted economic depressions. Section 3.6 critiques the application of the normative black swan theory in risk management, particularly beyond the short-term horizon.

Section 4 engages with Taleb's critique of finance as a pseudo-discipline. Using a simple one-period model, Section 4.1 illustrates the concept of the stochastic discount factor and critiques the alternative derivation of the Black–Scholes formula by Derman and Taleb (2005). Section 4.2 clarifies that the Nobel Prize awarded to Merton and Scholes recognized the groundbreaking idea of risk-neutral valuation, which catalyzed the entire field of martingale valuation in finance, providing a robust theoretical framework for deriving numerous fat-tailed option pricing models. Section 4.3 responds to Taleb's critique of Modern Portfolio Theory (Markowitz, 1952) and the Capital Asset Pricing Models (Sharpe, 1963; Merton, 1973a, 1973b), asserting that these seminal works facilitated the development of asset pricing models that accommodate fat-tailed, non-Gaussian distributions. Section 4.4 challenges Taleb's assertion that most variables in the social sciences follow scalable probability distributions.

Section 4.5 argues that finance's perceived pseudoscientific nature is more attributable to "games of manipulation" than lack of predictive power. Section 5 provides a summary and conclusions.

2 The Normative Black Swan Theory

2.1 Taxonomy of swans

Taleb (2007) articulates the concept of the black swan as follows:

(1) Remember that for an event to be a Black Swan, it does not just have to be rare, or just wild; it has to be unexpected, has to lie outside our tunnel of possibilities. You must be a sucker for it. (p. 212)
(2) If you know that the stock market can crash, as it did in 1987, then such an event is not a Black Swan. (p. 272)
(3) If you know that biotech companies can deliver a megablockbuster drug, bigger than all we've had so far, then it won't be a Black Swan, and you will not be surprised, should that drug appear. (p. 272)
(4) A black swan is about unknown unknowns. (p. 272)
(5) From the standpoint of the turkey, the non-feeding of the one thousand and first day is a Black Swan. For the butcher, it is not, since its occurrence is not unexpected. (p. 44)

These definitions illustrate that the black swan metaphor pertains to impactful events that are unforeseen from a subjective perspective. While the unexpected is occasionally conflated with the rare, not all rare and impactful events qualify as black swans. For instance, winning a hundred-million-dollar lottery, though rare and impactful, is not a black swan because it is a known unknown—an event anticipated with a small probability of a significant payoff. Black swans pertain to unknown unknowns, events whose probabilities are *erroneously presumed to be zero prior to their occurrence*. Table 1 classifies white swans, gray swans, and black swans based on the knowledge and type of their probability distributions. White swans are rare impactful events with known (or minimally erroneous) non-scalable probability distributions. Gray swans are rare impactful events with scalable, harder-to-estimate probability distributions. Black swans are impactful events erroneously presumed to have zero probability. While generally arising in scalable probability environments, black swan events do not need to be random; any impactful and unforeseen event from a subjective viewpoint

Table 1 Classification of Different Types of Swans.

Knowledge of the Probability Distribution	Type of Probability Distribution
	White Swan
Probability distribution is either known or estimated with small errors	Rare impactful events follow non-scalable distributions, such as a lottery win from a Gaussian distribution
	Gray Swan
Probability distribution is usually not easy to estimate	Rare impactful events follow scalable distributions such as Mandelbrotian power laws
	Black Swan
Probabilities are incorrectly assumed to be zero due to ignorance	Rare impactful events that are totally unexpected (e.g., equity market crash of 1987)

qualifies as a black swan, whether random or deterministic.

2.2 The five principles of black swan theory

With this taxonomy established, we can examine the normative black swan theory of investment decisions, as outlined by Taleb (2010):

(1) **Barbell Strategy:** "... your strategy is to be as hyperconservative and hyperaggressive as you can be instead of being mildly aggressive and conservative. Instead of putting your money in "medium risk" investments (how do you know it is medium risk? by listening to tenure-seeking "experts"), you need to put a portion, say 85 to 90 percent, in extremely safe instruments, like Treasury bills—as safe a class of instruments as you can manage to find on this planet. The remaining 10 to 15 percent you put in speculative bets, as leveraged as possible (like options), preferably venture capital-style portfolios." (p. 205)
"... Instead of having medium risk, you have high risk on one side and no risk on the other. The average will be medium risk but constitutes a positive exposure to the Black Swan." (pp. 205–6)

(2) **Diversification:** "Make sure that you have plenty of these small bets; avoid being blinded by the vividness of one single Black Swan. Have as many of these small bets as you can conceivably have. Even venture capital firms fall for the narrative fallacy with a few stories that "make sense" to them; they do not have as many bets as they should. If venture capital firms are profitable, it is not because of the stories they have in their heads, but because they are exposed to unplanned rare events." (p. 205)

(3) **Invest in Opportunities with Limited Loss, Without Requiring Precision:** "For your exposure to the positive Black Swan, you do not need to have any precise understanding of the structure of uncertainty. I find it hard to explain that when you have a very limited loss you need to get as aggressive, as speculative, and sometimes as "unreasonable" as you can be." (p. 207)
"... the scalability of real-life payoffs compared to lottery ones makes the payoff unlimited or of unknown limit." (p. 207)
"... here we do not know the rules and can benefit from this additional uncertainty, since it cannot hurt you and can only benefit you." (pp. 207–8)
"Likewise, do not try to predict precise Black Swans—it tends to make you more vulnerable to the ones you did not predict." (p. 208)
"Seize any opportunity or anything that looks like an opportunity." (p. 208)

(4) **The Irrelevance of Market Equilibrium:** "If you hear a "prominent" economist using the word *equilibrium*, or *normal distribution*, do not argue with him; just ignore him, or try to put a rat down his shirt." (p. 210)

(5) **Avoid Medium Risk Investments Exposed to Negative Black Swans:** "Learn to distinguish between those human undertakings in which the lack of predictability can be (or has been) extremely beneficial, and those where the failure to understand the future caused harm. There are both positive and negative black swans." (p. 206)
"Likewise, as we saw in Chapter 7, if you are in banking and lending, surprise outcomes are likely to be negative for you. You lend, and in the best of circumstances you get your loan back—but you may lose all of your money if the borrower defaults. In the event that the borrower enjoys great financial success, he is not likely to offer you an additional dividend." (pp. 206–7)
"My Scandal or Prediction (i.e., bogus predictions that seem to be there to satisfy psychological needs) is compounded by the Scandal of Debt. Borrowing makes

> you more vulnerable to forecast errors." (p. 314)
>
> "I worry far more about the "promising" stock market, particularly the "safe" blue chip stocks, than I do about speculative ventures—the former present invisible risks, the latter offer no surprises since you know how volatile they are and can limit your downside by investing smaller amounts."(p. 296)

The black swan theory underscores investor ignorance, advocating for investments in opportunities perceived as having zero subjective probability but with non-zero actual probability. For instance, investing in deep out-of-the-money put options, which might be valued at pennies but could appreciate to substantial sums if an unprecedented market crash occurs. The theory advises allocating 10–15% of a portfolio to highly diversified investments with positive black swan exposure (e.g., highly leveraged, speculative, aggressive, and opportunistic investments) and 85–90% to the safest Treasury bills, eschewing concerns for precision or market equilibrium dynamics. It also advises against "medium-risk" investments exposed to negative black swans, such as corporate loans, bonds, and stocks.

3 Fallacies Concerning the Normative Black Swan Theory

According to the five principles outlined in the previous section, the normative black swan theory is: (i) immune to significant losses in the short run, (ii) allows wild speculation in numerous opportunities, and (iii) leaves open the possibility of sudden and substantial gains. This has generated much excitement in the practitioner community regarding positive black swans. However, several questions remain unanswered: How does the normative black swan theory perform when the price for taking positive black swan exposure is very high (e.g., due to the option volatility smile)? Is this theory myopic, protecting only in the short run? Does this theory provide infrequent gains large enough to justify the continuous bleeding of the black swan portfolio? Is it easy or difficult to find positive black swan investments? Did the chase for positive black swans by the masses create financial bubbles such as the Tulip Mania, South Sea Bubble, Dot-com Bubble, and others throughout history? Can the avoidance of financial investments exposed to negative black swans, such as "medium-risk" corporate bonds and stocks, lead to economic depressions and significant shortfalls in investor wealth creation? Is the normative black swan theory useful as a risk management tool? The rest of this section addresses these questions by discussing six fallacies concerning the normative black swan theory.

3.1 When a smile kills your black swans

The first principle of the normative black swan theory requires an investment strategy of allocating 85–90% in the safest Treasury securities and 10–15% in speculative assets with exposure to positive black swans. A class of investable positive black swans includes deep out-of-the-money (OTM) options on stocks, bonds, currencies, interest rates, gold, silver, oil, electricity, and various other assets and variables, including a few non-tradables like weather and volatility. Typically, deep OTM options expire worthless unless an event causes the underlying asset to move sharply in the direction of the option exercise.

To illustrate how a positive black swan investor may benefit from deep OTM options, consider long-dated put options on the S&P 500 index (symbol: SPX) with strike prices ranging from 100 to 1200, as given in Table 1. The data provides closing bid and ask prices on November 4, 2011, for put options maturing on December 21, 2012. For example, consider buying a deep OTM

Table 2 Prices of European Options on S&P 500 and Implied Volatilities.

Strike	Bid price	Ask price	Black Scholes price	Ask price/ Black Scholes price	Implied volatility using ASK	Implied volatility using BIDASK	Open interest
100	$0.15	$0.20	$7.62E−27	2.62E+25	0.8346	0.8240	13,978
200	$0.60	$0.65	$2.99E−14	2.17E+13	0.6785	0.6753	15,226
300	$0.95	$2.35	$1.21E−08	1.95E+08	0.6242	0.5938	4,245
400	$3.70	$4.60	$1.92E−05	2.40E+05	0.5585	0.5488	23,047
500	$6.40	$8.30	$2.15E−03	3.87E+03	0.5084	0.4962	30,072
600	$10.00	$14.00	$0.05	260.34	0.4672	0.4506	85,267
700	$18.90	$22.90	$0.53	42.95	0.4345	0.4233	61,190
800	$27.90	$35.80	$2.87	12.49	0.4062	0.3901	38,869
900	$42.90	$50.00	$10.09	4.95	0.3693	0.3581	64,909
1,000	$64.20	$68.10	$26.33	2.59	0.3332	0.3282	67,888
1,100	$88.10	$96.00	$55.23	1.74	0.3078	0.2991	58,107
1,200	$123.40	$127.90	$98.65	1.30	0.2749	0.2704	113,055

Closing Date: November 4, 2011
Option Expiration Date: December 21, 2012
Underlying Asset Value: S&P 500 index Valued at 1253.23
Assumed Risk-free Rate: 0.12%
Assumed Dividend Yield: 2.01%
Historical Annualized Volatility (using daily returns from Nov 4, 2010 to Nov 4, 2011): 21.73%

put option with a strike price of 600, which is less than 50% of the S&P index value (closing at 1,253.23 on November 4, 2011). Taleb falsely conjectures that experts in socio-economic fields like finance predominantly use Gaussian models. Using the Gaussian Black–Scholes (1973) model with a continuously compounded riskless rate of 0.12% for 1.13 years (from November 4, 2011 to December 21, 2012), a continuously compounded dividend yield of 2.01%, and a historical volatility of 21.73% for S&P index returns (calculated from one year of historical daily returns), the Black–Scholes value of the put option is $0.0538, or slightly more than a nickel per option. Assuming the minimum tick size is $0.05 for SPX options, let us assume this option trades exactly for one tick or $0.05.[1]

In a hypothetical Gaussian world, consider investing $1,400 from a $100,000 black swan portfolio into such options. Since the notional value of each SPX option contract is 100 times the index value, each option contract is priced at $0.05 × 100 = $5.00, allowing the purchase of $1,400/$5.00 = 280 option contracts. Suppose the S&P index crashes to 500 by December 21, 2012. The initial $1,000 investment would multiply to 280 × (600 − 500) × 100 = $2,800,000, assuming option sellers would write such options at a tick in a Gaussian world. However, in reality, both academic consultants and option traders have been using non-Gaussian models since the 1987 stock market crash when the volatility smile first appeared. The Black–Scholes formula may still be used as an industry benchmark for communicating information about implied volatility, *but not for valuing options*. For the strike price of 600, far below the current index value of 1,253.23, the *implied volatility* based on the Black–Scholes model would be much higher due to the volatility smile. The actual closing ask price for this put option on November 4, 2011, was $14.00 (*about 280 times the Black–Scholes price of $0.05*), implying a volatility of about 46.72%. Consequently, a black swan investor could buy only $1,400/($14 × 100) = 1 option contract in the actual non-Gaussian world.

Even under a stark scenario where the S&P index crashes more than 60% from 1,253.23 to 500, the $1,400 investment in the put option would grow only to 1 × (600 − 500) × 100 = $10,000. If the $100,000 black swan portfolio had $85,000 invested in Treasury Bills earning 0.12% annually, $1,400 in the put option on the S&P index, and the remaining $13,600 in various deep OTM options in other market segments, the portfolio value would be $95,115.42 on December 21, 2012, despite a significant positive black swan event. In an even more severe scenario where the S&P index crashes to 400, the portfolio would be worth $105,115.42, a mere 5% gain from its initial value.

If some deep OTM options in other market segments, also expensively priced due to implied volatility smiles, ended up in-the-money, the portfolio might be worth more, say $150,000 or $200,000. However, these are the best-case scenarios of black swan events. The probability of any black swan event occurring such that at least one or more options pay off, is quite low, even using scalable Mandelbrotian power laws, making the expected gain from the black swan portfolio negligible and most likely negative. This is known as the "bleeding" strategy from investing in black swans.

If traders valued options naively as if the world were Gaussian, such bleeding could be justified, since a $100,000 black swan portfolio could grow significantly during black swan events. *However, financial professionals do not value options using Gaussian models.* Observing fat-tailed distributions in historical data, finance academics proposed non-Gaussian, jump-based models even before the 1987 equity market crash when the option volatility smile first appeared. For instance, Merton (1976) extended the Black–Scholes model to include sudden jumps of large magnitudes. His work, extended in numerous theoretical and empirical papers over the past three decades, covers various areas, including equity, interest rates, currencies, credit, and commodities.

Interestingly, Taleb (2007) does not cite any of these non-Gaussian option pricing models, creating a misleading impression that the finance field is stuck in Gaussian analysis. Many non-Gaussian options models are used routinely by financial participants either explicitly, or implicitly by fitting to the option smile.

This exercise demonstrates that deep OTM options do not have as much upside as Taleb claims. Option sellers are aware of potential black swan (or gray swan) investors and adjust implied volatilities accordingly. *Taleb confuses using the Black–Scholes model to communicate information about implied volatilities with using it for valuing options*. Since the 1987 equity market crash, no "marginal investor" has used the Black–Scholes model with historical volatility for option valuation; otherwise, the volatility smile would not have persisted.

The performance of the $100,000 black swan portfolio varies significantly depending on the volatility used: $2,885,115.42 using historical volatility[2] versus $95,115.42 with the actual 46.72% implied volatility on November 4, 2011. The more than doubling of implied volatility results in a 96.70% decrease in payoff, or $2,790,000, because the Vega (sensitivity of option price to volatility) of a deep OTM option is exponentially high. This exercise highlights the fallacies of Principles 1, 2, and 3 of the normative black swan theory. The upside potential of deep OTM options is determined by the relationship between the expected change in asset value at option expiration and the price of the option at the time of investment initiation. Taleb's (2010, p. 207) quote, "*I find it hard to explain that when you have a very limited loss you need to get as aggressive, as speculative, and sometimes as 'unreasonable' as you can be*," which forms the crux of Principle 3 stated in Section 2.2, is fallacious. Options will not make one wealthy if they are extremely expensive (by a factor of 280 times between the actual prices and the theoretical prices based on the Gaussian Black–Scholes model), even accounting for limited loss.

More evidence that investors and traders value options using non-Gaussian models comes directly from observing some of the columns of Table 1. Columns 2 and 3 give the actual bid and ask prices of options on the S&P 500 index for various strikes ranging from 100 to 1,200 on November 4, 2011. Column 4 provides the prices of these options using the Black–Scholes formula based on historical volatility. Since the Black–Scholes model relies on the Gaussian distribution, the prices this model generates for very low strikes converge to almost zero. For example, the option with a strike of 100 has a price with 26 zeros in front of the decimal, and the option with a strike of 200 has a price with 13 zeros in front of the decimal. This suggests the virtual impossibility of the S&P index dropping to 100 or 200 from its current level based on the Gaussian assumption.

However, participants in the option market using non-Gaussian distributions "expect" the index to fall to these low levels with non-negligible probabilities. There is significant "open interest" even at the strikes of 100 and 200 (see the last column of Table 1), and options for these strikes are priced at $0.20 and $0.65, respectively. Such prices can be obtained only by using non-Gaussian scalable probability distributions that allow for large and sudden jumps. Column 5 shows the ratio of the closing ask price of a given option to its Black–Scholes price based on historical volatility. For lower strikes, this ratio explodes to almost infinity, providing evidence that investors and traders are not using Gaussian models or any non-scalable cousins of the Gaussian models for pricing options. Columns 6 and 7 provide implied volatilities for various strikes using ask prices and the average of bid and ask prices, respectively, based on the Black–Scholes formula. Note that implied volatilities rise steadily from close to 27% for the strike of 1200 and exceed 80% for the strike of 100. Such volatility smiles are not new to the option markets and have existed since the equity market crash of 1987.

3.2 How market "equilibrium" creates negative black swans in the long run

A more significant problem with the normative black swan theory is that it is a *myopic*. Notwithstanding Principle 4 (which suggests irrelevance of market equilibrium), Principle 3 of this theory leads to endogenous creation of black swan "opportunities" by the suppliers of black swans that can cause negative black swan exposure for the investors in the long run. Though a black swan investor can never lose more than 10–15% of his wealth over a single period, the bleeding continues over time when speculative investments in expensive OTM options, new ventures, etc., expire worthless and the portfolio requires repeated allocations in such opportunities, until the unexpected happens! The investor experiences a negative black swan by losing much of the portfolio over years of chasing positive black swans—a remote possibility that the investor never considered due to ignorance, overconfidence, or both. Using the magic (or curse) of compounding, it does not take many years for the repeated 15% losses to drive a portfolio to a fraction of its original value.

In a free market ruled by the forces market equilibrium, the demand for positive black swans opportunities creates an equal supply of these "opportunities." Generally, the suppliers of investible positive black swans follow three strategies for benefitting from the buyers in the long run:

(i) charge extraordinarily high prices for positive black swan opportunities,
(ii) exaggerate the possibility of exponential gains of unknown limit, and
(iii) manipulate through fraud, scams, etc.

Unreasonably high valuations of deep OTM options are an example how sellers charge extraordinarily high prices for positive black swan opportunities.[3] As shown in the previous section, the growth of the investor portfolio with some deep OTM options can be anemic, *even in the period in which the black swan event occurs*, if one pays too high a price for these options. Furthermore, academic research shows that options sellers charge huge risk premiums for both the jump risk and the volatility risk, even *after* accounting for fat tails from non-Gaussian jump processes. Such risk premiums do not exist in the Gaussian Black–Scholes model, but are standard feature of all stochastic volatility jump models and arise due to sources of market incompleteness, that cannot be dynamically hedged away (e.g., see Pan, 2002). Many black swan investors do not realize that the 1987 equity crash was a great gift to the option writers. Even if some option writers lost over a single period, most options writers have made significant profits *over time* as the volatility skew experienced permanent upward shifts due to this event.[4]

The allure of positive black swans makes all of us to want a simple way to expose ourselves to these. However, most investors are neither lucky to father a Jeff Bezos—so that a small portion of the speculative part of their black swan portfolio invested in their son's venture grows to billions of dollars—nor blessed with great acting talent, voices, athletic abilities, and writing skills to become the rock stars of their professions. Most do not share a Stanford dorm room with the next dot.com celebrity or have millions in financial inheritance to become early angels of Google. The plain truth is that the most important black swans in our lives just happen to us (remember, they are "unexpected"), and we cannot go hunting for them. If there is systematic way to find "investible" positive black swans, then the race to get to these swans ensures that most are gone via huge and early increases in their prices. Investing at high initial prices hinders exponential gains even when black swan events are realized.

Furthermore, the problem of "silent evidence" is particularly problematic for black swan investors searching for an early investment in the next Facebook or the next biotech superstar. The failed ventures do not scream and shout about the dangers of speculation in the black swan territory, and the successful ones that everyone hears about do not sell ownership rights early enough for the masses to get rich. Perhaps, some venture capital funds can attract black swan investments from the ordinary folks. But the performance of such funds has been not much better than that of standard market indices (since these funds invest in a huge portfolio of new ventures, many of which do not perform well), and even if such funds would perform well temporarily, the huge demand from black swan investors would raise the price of investing in such funds to the point that they no longer remain "black swan" investments. This is similar to how the excess demand for deep OTM options raises the implied volatilities of these options enough that they no longer remain black swan investments.

In a market equilibrium, the clever sellers of black swan opportunities—aware of the problem of silent evidence for the buyers—exaggerate the possibility of exponential gains of unknown limit from these opportunities. Also, something more devious happens, as investors rush toward black swans. Many "me too" ventures and bogus companies are created and sold to gullible black swan investors who cannot know what is "fair and real," based on Principle 3 of this theory. For every venture that succeeds beyond one's imagination, there are numerous early start-ups that seem equally promising and fail. Some fail despite honest work, *but many fail because of sly effort*. Both the exaggerated possibility of exponential gains of unknown limit and outright scams by fraudulent companies explain at least partially how from time to time masses get overly excited and create sudden and temporary disequilibriums called "financial bubbles."

The bottom line is that the concepts of economic equilibrium (and disequilibrium) are relevant, *notwithstanding Principle 4 of the normative black swan theory*. An equilibrium in which everyone in the economy invests 10–15% in OTM options, new ventures, etc., is unstable since it is virtually impossible to invest 10–15% of the financial wealth of the entire economy in these "black swan" opportunities each year without creating financial bubbles. Moreover, an equilibrium in which everyone in the economy shuns investments exposed to negative black swans is also unstable. Yet, occasionally masses do get very afraid about investments exposed to negative black swans (such as the corporate bonds and stocks) and create disequilibriums, such as the great depression of 1930s. I turn to the topics of financial bubbles and economic depressions in Sections 3.3 and 3.5, respectively.

3.3 Do masses chasing positive black swans create financial bubbles?

The five principles underlying the normative black swan theory do formalize a system of thought that instinctively appeals to many investors. The somewhat contradictory goals of protecting most of one's assets, and at the same time speculating wildly with the remaining assets to become rich beyond one's dreams, explains at least partially why both insurance and gambling co-exist. More importantly, the insatiable hunger for positive black swans may also explain the occurrence of many financial bubbles in the human history, and can be traced back at least to the days of Tulip mania, if not earlier. Though the rare and precious tulips turned out to be worth little in the end, the early investors who sold out before February 1637 multiplied their investments many fold. The incredible rise (and the subsequent fall) in the Tulip prices was a classic positive (negative) black swan! As the gravity of economic equilibrium or a change in Dutch financial regulations that converted tulip futures into options brought the prices of tulips back to earth, many investors lost fortunes unable to find new investors to bail them out.

Given that some investors made fortunes, while many lost fortunes, it is not clear at first, whether the Tulip investors were "positive black swan investors" consistent with the five principles outlined in Section 2.2. However, note that according to the first three principles it is not the *timing* of entry into the tulip market but the *proportion* of one's portfolio exposed to the prices of tulips, which determines whether a simultaneously hyper-aggressive and hyper-conservative investor[5] is a positive black swan investor. If an investor had put only a small portion of the speculative part of one's portfolio in tulips, then such an investor was a positive black swan investor. It was not possible to know *ex-ante* if tulip prices could not keep multiplying to an "unknown limit." May be the rich and noble from all over Europe would also be smitten by tulips. May be the Nawabs, kings, and emperors from India, China, and other far away nations would sail to Holland in their quest to own these inimitable buds. From the perspective of a seventeenth century investor, putting a small portion of one's financial portfolio in tulips and similar investments would be filled with the enigma of "additional uncertainty," with the promise of an indefinite rich reward that could come in the end. A quote from Mackay (1841), says:

Every one imagined that the passion for tulips would last forever, and that the wealthy from every part of the world would send to Holland, and pay whatever prices were asked for them.

Since by Principle 3, "black swan investors make highly aggressive, speculative, and unreasonable investments, without having a precise understanding of the structure of uncertainty," such investors would not care, and perhaps not even know, if the entry into the market was too early or too late. The two important points to note are that:

(i) the price one pays for the investment, and
(ii) the probability of realization of the black swan event (it is "unexpected" by definition, so has seemingly zero probability)

are not what determines if one should invest in a positive black swan. The defining criterion is always "limited loss" exposure in highly scalable environments with potential for high payoffs if the positive black swan events are realized. Hence, a portfolio with a small investment facing a very limited loss in Tulips was a positive black swan investment, regardless of the point of entry in the market before it crashed. Such an investment satisfies all five principles of the normative black swan theory. Of course, if someone sold their whole farm to buy the most exalted *Admirael van der Eijck* bulb, this action would violate Principles 1 and 2 and so be inconsistent with this theory.

The normative black swan theory may also explain other financial bubbles in the recent human history. We can engage in thought experiments in which shipping companies that sent ships to the new world would benefit immensely from an exponential increase in the trade of slaves, raw materials, finished goods (and many other "unknown" items that could not have been imagined due to "additional uncertainty") between England and the American colonies. Positive black swans must surely be lurking in the minds of eighteenth century investors who bought the shares of South Sea and other shipping companies. The same could be said about many dubious companies during the French Mississippi bubble of 1719–20, the British railway mania of 1840s, the US railroad bubble of 1890s, and the dot.com bubble of late 1990s. In the early 1900s, the United States had more than 2,000 firms producing one or more cars. Many of these companies were in business for just one or two years, and by 1929 only 2% of these companies remained in business.

Where do we stop such pie-in-the-sky thinking—without the benefit of retrospective narratives that dismiss such thinking—if we follow Principle 3 of the normative black swan theory? Investing in positive black swans is the same old wine of purchasing the next powerball lotto in the new bottle, except in the former case investors embellish their hope by rationalizing their ignorance, and in the latter case they embellish it by exaggerating the odds. When masses express such wildly emotive behavior in the presence of a new source of additional uncertainty (e.g., a new technology, a new industry, and an emerging economy), it invariably causes financial bubbles and subsequent crashes, especially when they are dismissive about the relevance of economic equilibrium and disequilibrium (see Principle 4 in Section 2.2), and do not think such concepts have any bearing for investment decision-making. The normative black swan theory encapsulates the emotionally alluring, but somewhat naïve thought system practiced by masses during the turbulent financial periods in the recent centuries.

3.4 Positive black swans versus lottery tickets: Old wine in the new bottle?

Taleb suggests that collecting positive black swans is different from collecting lottery tickets, because lottery tickets are not scalable. *Is this really true?*

For example, consider following series of $1 lotteries that allow:

1/200 probability of winning $100,

1/200,000 probability of winning $100,000,

1/200,000,000 probability of winning 100,000,000, and so on.

Note that the ratio of increase in the lottery prizes is exactly the same as the ratio of decline in corresponding probabilities, giving a power law coefficient of 1.[6] For any size lottery, a single instance of "winning the prize" significantly affects the aggregate total won by all people buying that lottery. For any size lottery, it produces a few giants and millions of dwarfs. Obviously, lottery tickets are consistent with all of the properties of scalable laws (see Taleb, 2010).

Though lottery tickets are scalable, they are not *ex-ante* profitable. Scalability is about the shape of the return distribution, while *ex-ante* profitability is about the expectation of the return distribution. For example, each of the above lotteries is "expected" to make a loss of 50 cents or a 50% loss on the initial investment of $1. However, note that even *ex-ante* profitability does not explain the essential difference between the positive black swans and lottery tickets since by definition one cannot "know" the *ex-ante* profitability of a black swan investment.

Taleb argues that since black swans are about the unknown unknowns and lottery tickets are about the known unknowns, the upper limit on the returns of the former remains unknown, which can be used to distinguish them from the latter which have a known upper limit. An implicit assumption underlying this argument is that *the upper limit is higher if it remains unknown*, and so black swans benefit from this additional uncertainty. But in reality, the upper limit on the returns from some big lottery wins have been significantly higher than the upper limits on the returns on the best positive black swan investments in the entire human history. For example, the upper limit on the value of the winning ticket from a $1 lottery investment can be more than $100 million.[7] Assuming conservatively, that the winning ticket is announced one year after the purchase of the ticket, the upper limit translates into 10 billion percent return. A 10 billion percent return from a new venture company with $10k investment would mean growth of the company to $1 trillion *in a single year*. Do we really expect an upper limit of 10 billon percent return in a single year even for the venture capital firms that invested in companies like Google or Apple? For argument sake, Taleb might say that even a 10 billion percent upper bound on the realized return disqualifies an investment to be defined as a positive black swan, since the value of the upper limit is "known." Then how about raising the upper limit to say $10 trillion percent, or a quadrillion percent? So, what does the "unknown limit" really mean? Does the unknown limit mean "positive infinity" making the whole discussion absurd!

The discussion above is important because three separate ideas of: (i) scalability, (ii) *ex-ante* profitability, and (iii) size of upper limit, have been implicitly used by Taleb (2010) to distinguish between lottery tickets and positive black swans. For example, Taleb writes, quote,

> "Middlebrow thinkers sometimes make the analogy of such strategy with that of collecting "lottery tickets." It is plain wrong. First, lottery tickets do not have a scalable payoff; there is a known upper limit to what they can deliver. The ludic fallacy applies here—the scalability of real life payoffs compared to lottery ones makes the payoff unlimited or of unknown limit. Second, the lottery tickets have known rules and laboratory-style well-presented possibilities; here we do not know the rules and can benefit from this additional uncertainty, since it cannot hurt you and can only benefit you." (pp. 207–8)

Taleb mixes up the three distinct ideas of scalability, size of the upper limit, and *ex-ante* profitability (i.e., "expecting" to benefit from "additional uncertainty"). As shown earlier, *lotteries are scalable* even if they have a known upper limit. Furthermore, the known upper limit on the realized returns from the big lottery wins has been much higher than that of the best positive black swan investments in the human history. In other words, for all practical purposes, an extremely high value of the "known" can be higher than the unknown upper limit for some chosen phenomena even in extremistan. For example, it is extremely safe to say that the value of the next biotech star is not going to exceed $10 trillion next year. And finally, the "additional uncertainty" of the real world does not always benefit you. What if the "additional uncertainty" will bankrupt a new biotech firm (that one was hoping might produce the biggest blockbuster drug) with much higher odds? What if the "additional uncertainty" will make one's entire portfolio of black swan investments go close to zero over the next decade. Losing 15% each year in black swan investments over long periods means that, although additional uncertainty may not harm you in a single period, it can slowly bleed you to death over time.

Moreover, as the following example shows, an investor's portfolio can bleed to death even if the speculative part of the portfolio exposed to positive black swans is scalable, has infinite upside, and is highly profitable. Let us assume that the investor *does not know*, but the annual payoff from a black swan investment of $1 is *one* of the outcomes selected randomly from the following set of infinite possibilities:

1/10,000,000 probability of winning $100,000,000,

1/100,0000,000 probability of winning $1,000,000,000,

⋮

$1/10^N$ probability of winning $\$10^{N+1}$

(where the limit of N equals infinity)

The expected payoff from each of the above possibilities is $10. Hence expected annual return from this black swan investment (which is not known to the investor) of $1 is equal to $(\$10 - \$1)/\$1 \times 100 = 900\%$, making this investment far superior to purchasing lottery tickets. Furthermore, unlike the lottery tickets, the upper limit of this investment is *infinite*. And finally, the investment follows a scalable distribution with a power law coefficient of 1.

However, since the odds of winning are no more than 1 in 10,000,000 in any given year, even if an investor invests in hundreds of independent black swan investments that are similar to the above over many decades, the investor's portfolio would bleed to death over time with more than 99.9% probability. This example proves Taleb's conjecture wrong—*that only the impact, and not the probability of a positive black swan is important in extremistan.* One may lose only 15% of one's portfolio over one period, but almost 100% of the portfolio over one's entire life, even when payoffs are hugely profitable, scalable, and have an infinite upper limit!

3.5 Can avoiding "medium risk" investments exposed to negative black swans (i.e., the corporate bond and stock markets) lead to economic depressions?

Principles 1 and 5 of the normative black swan theory recommend zero exposure to "medium-risk" investments exposed to negative blacks swans. Taleb classifies small portfolio allocations in speculative investments such as options and new ventures as positive black swan investments and large portfolio allocations in medium-risk investments such as corporate bonds and stocks as negative black swan investments. This distinction is as arbitrary as it is inadequate. It is well known that due to the limited liability of corporate owners, a corporate stock is equivalent to a call option on the underlying assets of a corporation. Also, the return from a deep in-the-money (ITM) call option is similar to the return on the asset underlying the option. In other words, deep ITM call options on blue chip stocks are medium-risk investments, and penny "stocks" of almost bankrupt companies are equivalent to high-risk deep OTM call options.[8] Hence, black swan investments cannot be always defined as either "positive" or "negative" based on security or asset type.

Furthermore, the risk in corporate securities can be either amplified or diluted, depending upon the implicit leverage of the options embedded in these securities with respect to the underlying asset. A corporation with extremely low leverage ratio of say 5% is not going to default on its debt if it has moderate business risk, as bondholders have enough cushion against the downside. The default put option embedded in the debt issued by such a corporation is very deep out-of-the-money, and so the corporate debt behaves almost like a Treasury security with virtually no exposure to negative black swans. In fact, an asset class that performed exceedingly well during the Great Depression was high-quality investment grade bonds whose yields actually went down and prices went up from 1930 to 1933,[9] a period when the stock market lost much of its value. Lower rated bonds did not do well, of course, and many of these defaulted.

Just like its foolhardy to buy expensive put options in the presence of a steep implied volatility smile in the hope of a market crash (see Section 3.1), it is unwise to get rid of very high-quality investment grade corporate bonds due to the fear of negative black swans. The increase in the value of the high-quality investment grade bond due to a rise in its long "Treasury component" (due to a decline in Treasury rates) would most likely offset the decrease in the value of this bond due to a rise in its short "put option component" in the aftermath of a market crash. The reverse is true for low-quality junk bonds. To see this, consider the following example of a triple-A rated corporate bond that is priced at \$100, as follows:

$$\text{Corporate Bond} = \text{Treasury Component MINUS}$$
$$\text{Default Put Option},$$
$$\$100 = \$100.45 - \$0.45$$

Assume that due to a negative black swan event, the corporate bond price goes up to \$101.00, as follows:

$$\$101 = \$105.45 - \$4.50$$

Note that the default put option multiplies 10 times from \$0.45 to \$4.50, due to the negative black swan event, but due to an increase of \$5.00 in the Treasury component (assuming the duration of the bond is close to 5 years, and a decline of 100 basis points in 5-year Treasury yield), the value of corporate bond increases by $1. This example explains why high-quality investment grade corporate bonds have done well historically, including during the Great Depression of 1930s, in the aftermath if the 1987 equity market crash, and more recently in the aftermath of the credit crisis of 2007–2009.[10]

However, suppose that investors sold off their high-quality investment grade bonds at the onset of these financial crises. Needless to say the Great Depression would have lasted a lot longer, and perhaps the western economies may have *never* gotten out of the depression if investors never purchased corporate bonds again. And in case these economies did get out of that depression by investors getting back into corporate bonds, getting rid of them again in 2007 or 2008 would have most definitely thrown us back into the mother of all depressions as the economic conditions surrounding this crisis were far worse than those prevailing in 1929.

A total paranoia against negative black swans in one's life would mean one does not engage in any risky activities—including crossing the street or driving a car—causing a psychological depression, disease, or even death. Similarly, avoiding *all* medium-risk investments such as corporate bonds and stocks would either cause a permanent depression in the economic body, or replace the free-market capitalist system with some sort of a communist rule in which government would take over most of the economic activities (except new private ventures, and options on new ventures) by issuing tens of trillions of additional government bonds and using the proceeds to buy up all corporate bonds and stocks—since 85% of all investors wealth must be invested in government securities by Principle 1 of the normative black swan theory.

Avoiding corporate bonds and stocks also has dire implications for the growth of investors' portfolios over long horizons. Consider how much $10k will grow to in 50 years if invested in any of the following:

	After 50 years
T-bills – Offering 3%	$44 k
T-bonds – Offering 6%	$189 k
Corporate bonds – Offering 8%	$469 k
Corporate Stocks – Offering 12%	$2.89 million
Genius investor – 24%	$468.9 million

An investor putting all of her portfolio in Treasury bills would have less than $45k versus $2.89 million for an investor putting all her money in corporate stocks. This is the magic of compounding—even though the return of 12% is only four times the return of 3%, the wealth of the stock investor grows to *66* times the wealth of the Treasury bill investor. And even though the return of 24% is only twice the return of 12%, the wealth of the genius investor grows to more than *162* times the wealth of the stock investor. Even if the genius investor lost 99% of his wealth after 50 years, he would still have $4.689 million left, or more than 100 times the terminal wealth of the T-bill investor. The magic of compounding of "medium-risk" investments explains how Warren Buffett became one of the richest people on earth, starting with only $10,000 in early 1950s.

If the normative black swan theory produced outsized returns over long periods through 10–15% portfolio exposure to positive black swans (and 85% exposure to Treasury bills), then we would see many genius black swan type investors who dabbled in new ventures, options, etc. History has not found any such hyper-conservative/hyper-aggressive billionaire investors who keep most of their money in short-term Treasury securities and dabble with only a small portion in new ventures, options, etc. New ventures are marvelous for the owners of the ventures, *who put 100% their money in their ventures without any diversification* (violating both Principles 1 and 2 of the normative black swan theory). But almost no "smart and ordinary investors" have become super rich by investing only 10–15% of their portfolios in a diversified set of hard-to-find positive black swan investments.

3.6 The illusion of financial risk management using the normative black swan theory

Can the normative black swan theory be used for managing financial risk? Consider the following implications of this theory:

(1) *Normative black swan theory is inconsistent with market equilibrium.* Investing 85–90% of one's portfolio in the safest US Treasury securities, 10–15% in the highly risky options, venture capital, etc. (exposed to positive black swans), and 0% in the medium-risk corporate bonds and stocks (exposed to negative black swans) by everyone implies an unsustainable market equilibrium in which publicly traded corporations would disappear.

(2) *Normative black swan theory is myopic.* As shown in Sections 3.1 and 3.2, chasing positive black swans based on Principle 3 can lead to huge losses over the long run due to "bleeding" in option markets with high implied volatilities, and losses in dubious ventures.

(3) *Normative black swan theory creates financial bubbles*. As shown in Section 3.3, as a huge number of investors rush into new ventures/industries to capture opportunities of unlimited gains with limited losses (i.e., "highly aggressive, speculative, unreasonable opportunities, or anything that looks like an opportunity... without requiring any precision" as stated in Principle 3), this theory can create financial bubbles and busts.

(4) *Normative black swan theory leads to economic depressions.* As shown in Section 3.4, avoiding investments in the corporate bond and stock markets due to a total paranoia of negative black swans will push an economy into a depression or a communist style takeover of the economy.

The normative black swan theory is only good for risk management of a *single agent over a short horizon*. It can neither be used for managing risk in the long run (due to bleeding) nor used for understanding systemic risk caused by interaction of multiple agents in the entire economy (since it considers economic equilibrium irrelevant, and leads to financial bubbles and economic depressions). While the above discussion does not dispute the relevance of the "positive" black swan theory for explaining reality, it demonstrates the limitation of the "normative" black swan theory for *managing financial risk*. In fact, not all concepts that explain reality well are useful for managing reality. For example, knowing that a butterfly flapping its wings in New Delhi can create a hurricane in New York does not prepare New Yorkers any better in facing the wrath of the hurricane. In this important respect, the black swan concept is not so different from the butterfly effect—both are useful for explaining, but not so useful in predicting and managing risk.

4 Is Finance a Pseudo Discipline?

4.1 The "incorrect" derivation of Black–Scholes formula by Derman and Taleb

Arguably, the most celebrated result in the field of investments is the first fundamental theorem of asset pricing, which gave rise to the subfield of martingale pricing. According to this theorem, the "absence of arbitrage guarantees the existence of an equivalent martingale measure under which all discounted asset prices are martingales" (see Harrison and Kreps, 1979; Harrison and Pliska, 1981; among others). The equivalent martingale measure is a pseudo-probability measure under which all discounted prices are martingales. Martingale pricing does not require complete markets, continuous-time trading, dynamic hedging, or strong restrictions on asset return distributions and utility functions of investors. The only major assumption needed for this theorem to apply is the absence of arbitrage.

Martingale pricing theory has been recently generalized by Nawalkha and Zhuo (2022) to equivalent expectations measures (EEM) theory. This new framework allows for the derivation of analytical solutions for the expected future prices or expected returns over a finite horizon for all contingent claims that admit an analytical solution to their current price using martingale pricing. Although Black and Scholes (1973) derived the formula for a call option using a partial differential equation, this formula can also be derived using martingale pricing, and the expected return of the call option can be derived using EEM theory.

It would not be an exaggeration to say that the first fundamental theorem of asset pricing is to finance what the first law of thermodynamics is to physical chemistry. Just as new energy cannot be created but only transformed from existing energy, riskless arbitrage profits cannot be created by transforming existing wealth. Taleb (2007) discusses the limitations of Gaussian distributions and highlights the importance of fractal and power law models, pioneered by Benoit Mandelbrot. He contrasts these models with traditional financial models that rely on Gaussian distributions. It is noteworthy that it is not the Gaussian distributions but the “absence of arbitrage” argument put forth by the Nobel papers of Black and Scholes (1973) and Merton (1973a, 1973b) that eventually led to the development of the first fundamental theorem of asset pricing. Even Mandelbrotian power law returns must obey the first fundamental theorem of asset pricing to prevent riskless arbitrage. This theorem applies in incomplete markets also where returns may follow any arbitrary discrete or continuous-time distributions, including stochastic volatility, compound-Poisson jumps, general Levy jumps, and other return characteristics. Furthermore, this theorem establishes the equivalence between the absence of arbitrage and the existence of a positive stochastic discount factor that prices all assets, as shown by Harrison and Kreps (1979), Hansen and Richard (1987), and Hansen and Jagannathan (1991).

Minimizing the contribution of the above stream of research, Derman and Taleb (2005) falsely claimed that the basic put-call parity and the forward-spot relation is sufficient for the derivation of the Black and Scholes option formula, *and the arbitrage-free pricing argument used by Black and Scholes and martingale pricing are redundant*. To understand the basic error made by Derman and Taleb, consider the intuition behind the use of the stochastic discount factor for valuation. The stochastic discount factor—which must exist and be positive to prevent riskless arbitrage by the first fundamental theorem of asset pricing—allows using different discount rates to discount cash flows that occur in different states. As an example, consider an economy over a single period in which only three states can be realized with equal likelihood—a state in which the economy booms, a state in which the economy is normal, and a state in which the economy goes bust. Now, consider the end-of-the-period payoffs from two securities such that both securities pay \$100 on an *expected* basis, but the first pays \$150 in the boom state, \$100 in the normal state, and \$50 in the bust state, and the second pays \$350 in the boom state, \$50 in the normal state, and −\$100 in the bust state. Which security is worth more today? If investors are risk-averse, they would be worried about their job security and consumption, more in the bust state than in the boom state. Thus, a dollar in the bust state would be *discounted less* than a dollar in the boom state, because the dollar in the bust state is "more valuable" than a dollar in the boom state. Without loss of generality, assume that investors apply a 5% discount rate under the bust state, a 10% discount rate under the normal state, and a 15% discount rate under the boom state. The prices of the two securities are computed as follows:

First Security Price:

$$P_1 = \frac{1}{3} \times \frac{\$150}{1.15} + \frac{1}{3} \times \frac{\$100}{1.1} + \frac{1}{3} \times \frac{\$50}{1.05}$$
$$= \$89.654$$

Second Security Price:

$$P_2 = \frac{1}{3} \times \frac{\$350}{1.15} + \frac{1}{3} \times \frac{\$50}{1.1} + \frac{1}{3} \times \frac{-\$100}{1.05}$$
$$= \$84.855$$

So, even though both securities have equal expected cash flow of $100 at the end of the period, the first security is valued more today, because the first security delivers a positive cash flow in the bust state, while the second security delivers a negative cash flow in the bust state. Using the standard textbook approach, the "expected" cash flow of $100 of the first security must be discounted *less* using a "constant" discount rate because its present value is *higher*.

The textbook method would compute the constant discount rate for the first security as follows:

$$\begin{aligned} P_1 &= E(\mathrm{CF}_1)/(1 + R_1) \\ &= \$100/(1 + R_1) \\ &= \$89.65, \quad \text{hence}\, R_1 = 11.54\% \end{aligned}$$

Similarly, the textbook method would compute the implied constant discount rate for the second security as:

$$\begin{aligned} P_2 &= E(\mathrm{CF}_2)/(1 + R_2) \\ &= \$100/(1 + R_2) \\ &= \$84.86, \quad \text{hence } R_2 = 17.85\% \end{aligned}$$

The implied constant discount rate for the first security is lower by more than 6%. This is because the first security is more useful for hedging against an economic bust when one is more likely to be unemployed and have suffered portfolio losses. In the asset pricing literature, the first security would be associated with lower "systematic risk" and hence, would require a lower risk premium by investors. The fact that one uses "constant" discount rates does not mean that the concept of stochastic discount rate has no relevance. *The very reason the implied constant discount rate is lower for the first security is because using the stochastic discount rate implies that the dollar received in the economic bust state is worth more today than the dollar received in the economic boom state.*

Note that we have not invoked the Gaussian assumption or dynamic hedging to make this argument based on economics of risk aversion (i.e., investors are more concerned about their utility in the bust state than the boom state). The argument holds even in incomplete markets. However, Derman and Taleb (2005) claim that the concept of stochastic discount rate is irrelevant, and provide a derivation of the Black–Scholes formula, *which does not allow the discount rate to be different in different states*. If the discount rates are identical for all states, then by definition investors are in a *risk-neutral world*, and all expected cash flows must be discounted by the riskless rate. But, in the above example, it is impossible for any specific value of riskless rate to simultaneously obtain the price of $89.654 for the first security and $84.855 for the second security, if the expected cash flow from both securities is $100. Hence, Derman and Taleb's derivation is inconsistent with how securities are priced under the physical measure. Since risk-neutrality is not an *assumption* but an economic implication in the Black and Scholes (1973) paper—which is consistent with a stochastic discount factor for pricing the assets under the physical measure—their derivation of the formula represents a major breakthrough in option pricing. On the other hand, since risk-neutrality is an *assumption* for the derivation of Black–Scholes option pricing formula by Derman and Taleb (2005), their derivation of the formula is incorrect and valid only when all investors have risk-neutral preferences.

Though for expositional purpose, the above example assumed a complete market (i.e., in which each state of the world can be spanned by portfolios of existing securities), the first fundamental theorem of asset pricing continues to hold under all sources of market incompleteness including arbitrary probability distributions in discrete time. Though under incomplete markets, the risk-neutral probabilities cannot be uniquely determined,[11] they can be obtained by either calibrating to the cross-section of option prices or estimating the appropriate stochastic discount factor using the historical data, and then using a change of probability measure.[12]

4.2 A plethora of non-Gaussian option pricing models in finance

A casual reading of *The Black Swan* would make it seem that academic work on option valuation is based solely on the Gaussian assumption with few insignificant extensions since the book does not cite the numerous papers that routinely use non-Gaussian assumptions from a very large body of theoretical, empirical (both economic and methodological), and numerical literature on options models based on compound-Poisson processes and Levy processes. In fact, academics have produced more non-Gaussian option pricing models than Gaussian models since the equity market crash of 1987. In his foreword to Gatheral's (2006) *The Volatility Surface*, Taleb wrote, quote,

> "I recently discovered the strength of his (Jim Gatheral's) thinking as follows. When, by the fifth or so lecture series I realized that the world needed Mandelbrot-style power-law or scalable distributions, I found that the models he proposed of fudging the volatility surface was compatible with these models. How? You just need to raise the volatility of out-of-the-money options in a specific way, and the volatility surface becomes consistent with the scalable power laws."

Jim Gatheral recommends stochastic volatility jump (SVJ) models and Levy jump models based on compound-Poisson processes and the more general Levy processes, respectively. The first such model was invented by none other than the Nobel Laureate Robert C. Merton (1976) who was fully aware of the limitation of the Black–Scholes–Merton formula for option pricing in the presence of large and sudden jumps in the stock price movements. He added a compound-Poisson jump component to the diffusion process in order to allow fat tails that cause "six-sigma" events. Since then numerous theoretical, empirical, and econometric studies have extended his model, and investigated the properties of the SVJ and the Levy jump models that allow non-Gaussian distributions. None of these fat-tailed models are cited by Taleb (2007, 2010) giving a false impression that finance field is stuck in Gaussian analysis.[13]

The Nobel prize given to Merton and Scholes for their work on options should be viewed in the larger context of option pricing based on Gaussian as well as non-Gaussian models. The fundamental contribution of these authors is not the Gaussian formula, but the implicit idea of "risk-neutral valuation," which is based upon absence of arbitrage. Further, the initial work of Black and Scholes (1973), Merton (1973a, 1973b), and Cox and Ross (1976) led to the development of martingale valuation theory that significantly generalized the idea of risk-neutral valuation to *incomplete markets* using the first fundamental theorem of asset pricing (see Harrison and Kreps, 1979; Hansen and Richard, 1987; Hansen and Jagannathan, 1991). This theorem provided the appropriate framework for the derivation of various stochastic volatility jump models and Levy jump models in incomplete markets. This theorem does not require continuous-time trading or dynamic hedging and applies even to power law distributed returns, unless one admits the possibility of riskless arbitrage. Hence, the origins of the fat-tailed incomplete market option pricing models can be traced all the way back to the idea of "risk-neutral valuation" introduced in the Gaussian models of Black–Scholes and Merton. The Nobel was given not for the Gaussian option formulas, but for the path breaking idea of "risk-neutral valuation" that spawned the entire field of martingale valuation in finance.

4.3 Other Nobels in finance

The Black Swan might convince a non-finance reader that Modern Portfolio Theory (Markowitz, 1952), the Capital Asset Pricing Model (Sharpe, 1963; Merton, 1973b), and the Gaussian option formulas (Black and Scholes, 1973; Merton, 1973a), represent the main theories that define modern finance, and that nothing of major significance, except some minor extensions, has been achieved after Markowitz and Sharpe not because of the Gaussian models they invented, but because these models led to fundamental insights about risk and return, setting the stage for the development of a whole range of asset pricing models that allow non-Gaussian distributions. The main insight contained in Markowitz's modern portfolio theory is that *diversification can lead to more efficient allocation of risk and return*, i.e., holding a large number of securities from many asset classes (say, equities, bonds, gold, oil, real estate, emerging markets, etc.) can reduce the overall risk of one's portfolio. This insight continues to apply in virtually all non-Gaussian settings, including even under scalable power laws. For example, a mutual fund typically has lower risk than a single stock, and a portfolio of mutual funds invested in various asset classes and countries has lower risk than one specialized mutual fund. This insight has been used in practical contexts by numerous institutional investors and ordinary investors for achieving diversification. For example, returns on precious metals, Treasury bonds, and equities, have moved quite independently, and sometimes even with negative correlations during the worst financial crises in the recent past. Also, Principle 2 of the Black Swan Theory (see Section 2.2) is based explicitly on the idea of diversification, and some of the newly created Black swan funds use this insight for efficient allocation of risk and return. Though Markowitz presented his work using the mean/variance framework, extension of his work allows using the diversification principle under significantly weaker distributional assumptions. For example, Chamberlain and Rothschild (1983) and Ingersoll (1984) extend the idea of optimal diversification to include arbitrary return distributions under Ross's (1976) APT using the concept of a *well-diversified portfolio*.

Similarly, the main insight contained in the Sharpe's CAPM is that only the *systematic risk of an individual security is priced*. This insight continues to hold under all non-Gaussian extensions of the CAPM including the co-skewness based CAPMs of Kraus and Litzenberger (1976), Harvey and Siddique (2000), and Smith (2003); the APT of Ross (1976), the equilibrium APT of Connor (1984), and the MFST of Ross (1978), *all with arbitrary return distributions*; and numerous stochastic discount factor-based models such as the non-linear pricing kernel model of Dittmar (2002), and the non-linear APT models of Bansal and Viswanathan (1993) and Nawalkha (1997).

It is inconceivable that the more general results in numerous extensions of the models of Markowitz and Sharpe would have come easily, without the prior work done by these Nobel Laureates on the (i) efficient allocation of risk and return using diversification and (ii) pricing of systematic risk, respectively. Knowledge builds sequentially in any field, and seminal papers that lead to the development of entire new paradigms with thousands of citations are worthy of the prize.

4.4 Extremistan or extreme reductionism?

Taleb classifies entire academic disciplines using the metaphor of extremistan and mediocristan. The variables in extremistan follow scalable probability distributions and are highly susceptible to black swans. Though variables in mediocristan may also allow an occasional black swan (generally based on subjective ignorance), usually these variables allow only white swans from non-scalable probability distributions. Taleb claims that most variables in social sciences like economics and finance follow scalable probability distributions of extremistan. Taleb's (2007, p. 35) list of economic variables from extremistan includes everything from "wealth, income, sizes of companies, stock ownership, financial markets, commodity prices, inflation rates, and economics data."

While it may be true that distributions of many micro-economic variables such as individual company sizes and individual stock ownership belong to extremistan, it seems somewhat of an exaggeration to suggest that most macro-variables follow scalable distributions. For example, consider the GDP growth rate over the past 100 years. This variable is not going to suddenly jump to 100%, especially for large economies like those of United States and China. Though the founders of Google or Facebook may find their net worth grow exponentially from a few thousand dollars to a few billion dollars, entire nations cannot grow at such rates because of the diversification and redistribution effects over a large number of units. As some successful firms grow at a very rapid pace, resources from other firms get redistributed to the successful ones through the invisible hand of free markets. In general, if one were to investigate the *intertemporal behavior* of many macroeconomic variables, one may find that many such variables can be modeled in a non-scalable environment. For example, consider the economic objective of increasing the median per capita GDP and/or to reduce the volatility of this variable over time. Modeling the intertemporal behavior of this variable using a non-scalable probability distribution may be totally legitimate, as it follows the "supreme law of mediocristan," which states (see Taleb, 2007, p. 32):

> "When your sample is large, no single instance will significantly change the aggregate or the total."

Since a single-quarter growth rate in median per capita GDP will remain an insignificant part of the total growth in this variable over say 50 or 100 years, this variable belongs to mediocristan. This is true despite the fact that at the micro level the wealth of individuals like Mark Zuckerberg may grow many fold over a single day. In other words, mediocristan and extremistan can co-exist in the same economic reality. Such co-existence of mediocristan and extremistan occurs in natural sciences as well. For example, the probability distribution of an individual sperm fertilizing an egg may belong to extremistan, while the probability distribution of the number of children per family may belong to mediocristan. Just like at the *macro level* one would not use scalable power laws to model the number of children per family in a region or a country despite the fact that at the *micro level* only one sperm wins the race from more than 500 million, one would not use scalable power laws to explain the intertemporal variations in the GDP growth rate of a large developed economy like that of United States, despite the fact that the wealth of a specific individual may suddenly rise or fall many fold over a single day.

In a recent critique of Taleb (2007), Treynor (2011) questions whether diversified stock indices belong to extremistan when viewed from the perspective of a long-term investor using *annualized log returns*. Treynor shows that even if daily or weekly log returns have significant fat tails—which would scare short-term traders—time diversification using the central limit theorem, which does not require the underlying daily or weekly log return distributions to be Gaussian, makes annual log returns to have almost a Gaussian distribution over the past 200 years. Hence, it is important to distinguish investors as either "short-term traders" or "long-term investors" to assess the dangers of negative black swans. Short-term traders may fear common stocks due to the occurrence of "six-sigma" events over a given day or a week, but long-term investors could remain steadfast in their love of common stocks due to diversification over time. For example, though the 1987 equity market crash was a frightful negative black swan on Monday, October 19, when the S&P 500 index lost 20.47% of its value, the annual decline of only 6.20% in this index from January 1987 to January 1988 was hardly noticed by long-term investors. Similarly, the 40% decline in the S&P 500 index from January 2008 to January 2009 due to the Lehman bankruptcy and other related events represents only a two-sigma event over the annual horizon, using a 20% annualized historical volatility of the index returns. Clearly, one does not need Mandelbrotian power laws when investors use annualized log returns for performance evaluation.

Taleb and Martin (2012) respond to Treynor's critique by noting that a power law distribution could generate returns that *appear* Gaussian due to small sample size effects. However, a similar line of reasoning would invalidate the use of power law distributions, as well. As shown by Weron (2001), it is virtually impossible to know whether the power law coefficient is 1.8 or 2.7 or 3, due to small sample size effects. Since the tail is never easy to estimate using limited amount of financial data, the disagreement between Treynor (2011) and Taleb and Martin (2012) cannot be resolved using statistical techniques, and Bayesian beliefs may have to be used instead.

Taleb and Martin further claim that:

> "The results of Markowitz cannot accommodate power laws, finite or infinite variance (though derivatives pricing is not affected at all by such argument)."

The statement within the parenthesis is not true. If one cannot know the power law coefficient due to small sample size effects (see Weron, 2001), then how can one price derivatives using power laws? Basically, Taleb and Martin's (2012) criticism of Treynor is nihilistic; it leads to the implication that quantitative models cannot be used for portfolio diversification or for derivative pricing with the *finite size* of data typically used in finance. The only way to go beyond such nihilism is to use Bayesian beliefs regarding the underlying statistical distributions. For example, Taleb may believe that US Treasury interest rates follow a power law distribution based on his analysis of other western countries that have experienced hyperinflation. Another investor like Warren Buffett, who has more confidence in the size of US economy (and US military), as well as in the independence of the US Federal Reserve from the US government, may believe that US Treasury rates follow a mixed jump-diffusion process. Hence, purely statistical arguments cannot determine whether a given macro-variable is from extremistan or mediocristan.

4.5 Why has finance become pseudo: Scandal of prediction or manipulation of information?

Consider a thought experiment in which Dilbert and Catbert play a game in which they reveal the surfaces of two coins, simultaneously. If both reveal HEADS, Dilbert wins a dollar, and if both reveal TAILS, Dilbert wins a dime. If they reveal unmatched surfaces, Catbert wins 50 cents. Nassim Taleb has been asked to be the financial expert who must predict which surface will be revealed by which player to win this game.

Though "pure strategy" prediction (i.e., predicting either HEADS or TAILS by each player) is impossible in this game, suppose Taleb predicts that Catbert would play HEADS. Based on Taleb's prediction, Dilbert would also play HEADS to win $1. But if Dilbert played HEADS, then Catbert would play TAILS to win 50 cents. But if Catbert played TAILS, then Dilbert would change his mind and play TAILS to win 10 cents. Finally, if Dilbert played TAILS, then Catbert would also change his mind and play heads to win 50 cents... or back to square one without either player finding a pure winning strategy.

Predicting a pure strategy in this two-person, non-cooperative, zero-sum game is not possible because the optimal solution is to keep the other person *guessing*, similar to how a tennis player will keep her opponent guessing about the direction of her serve. Suppose Taleb's expertise in finance was to be judged by his prediction about which surface will be revealed by which player in each instance they play—with 95% confidence interval. Then Taleb would most likely be judged as a non-expert! The moral of this game is that in many instances, the inability to predict an outcome is both *optimal* and *rational.*[14]

Hence, before judging entire disciplines based on predictability, one should consider whether the phenomena at hand should be predictable or not in a rational equilibrium. Should stock prices be predictable? If so, then who in their right minds would sell (buy) if the stock markets were about to rise (fall) sharply? Should analysts be able to predict earnings with higher accuracy, consistently? If so, then why would these analysts continue to make predictions to make others rich instead of starting their own hedge funds? What if a central bank could predict that inflation was going to be in double digits after five years? Wouldn't such a prediction make the central bank take powerful actions to combat inflation, preventing the inflation from occurring? While it is true that most economists did not predict the financial crisis of 2007–2009, the monetary and fiscal responses by the US Federal Reserve and the US Treasury, respectively, have been extremely effective in saving United States and the rest of the world from the mother of all depressions. If the expertise of a medical doctor is judged not by the ability to predict, but by the skill in detecting and curing a disease, why should we judge Ben Bernanke's expertise by his inability to predict the credit crisis, instead of the effectiveness of his policies since 2008.

Notwithstanding the above analysis, Taleb classifies entire disciplines as real or pseudo based on the "ability to predict." Pure disciplines such as plumbing or chemical engineering are real because they can predict outcomes with high accuracy, while social disciplines like finance, economics, political theory, etc., are pseudo because they have a dismal record at predicting. However, Taleb ignores an important quantum-like aspect of social disciplines—the act of prediction by the observer influences the observed reality in these disciplines. Furthermore, social disciplines are about interaction between living entities, and in many instances *non-prediction is a rational outcome* when the goal is to keep the opponent guessing as in the game between Dilbert and Catbert given above.

This mixed strategy solution to the two-person, non-cooperative, zero-sum game (e.g., like the game between Dilbert and Catbert)[15] launched game theory as a scientific discipline in 1928, when Jon Von Neumann—who many believed was the smarter of the two Princeton geniuses of twentieth century—derived the *Minimax theorem*. Since then game theory has been generalized to a variety of settings under significantly weaker assumptions for studying interaction between two or more agents, such as humans, animals, plants, and bacteria. Applications have been made in numerous fields including, economics, business, politics, psychology, biology, zoology, evolution, sports, logic, computer science, philosophy, and others.

Unlike the fictitious game between Dilbert and Catbert, real-life games do not always have "optimal" outcomes when everyone maximizes their short-term gain in a world with imperfect information. For example, all players are worse off when they choose the Nash equilibrium in a Prisoner's dilemma game (i.e., the cheating strategy). As a recent example, the credit crisis of 2007–2009 can be viewed as an incomplete information prisoner's dilemma game between bank CEOs in which each CEO wanted to increase his/her bonus package that was based on short-term earnings of the bank. An easy way to increase the short-term earning for a bad bank CEO was to secretly reduce the quality of loans to get higher yields in the short term, followed by getting cheap insurance through the purchase of credit default swaps (CDS) on the bad loans on the book, and/or selling these bad loans to gullible investors by manipulating information via securitizations (while sacrificing long-term reputation of the bank due to CEO's myopic incentives), and write CDS on the loans of other banks assuming that other bank CEOs must be behaving as good citizens and issuing high-quality loans.

If these assumptions held correctly, this strategy would increase the short-term earnings, and give the bad bank CEO a bigger end-of-the-year bonus. Assuming that over time other bank CEOs figured out this strategy, as well, at some point all banks would become bad, and then gradually the fictitious increases in earnings would evaporate for all, leading to the cheating Nash equilibrium. Of course, the myopically incentivized CEOs would retire with larger than life bonuses or be fired with huge golden handshakes when all the bad news is revealed to the bank shareholders. Furthermore, if the CEO of some non-bank financial institution (e.g., AIG) ignorant about the multiple-agent Prisoner's dilemma being played by the bank CEOs, also desired to increase his bonus through short-term earnings, then he would be the last fool manipulated into writing cheap insurance due to the information asymmetry between banks and non-banks.[16] Hence, it may not be the "scandal of prediction" which makes finance a pseudo discipline, but crafty manipulation by bank CEOs through packaging of dubious loans into CDOs, purchases of CDS to protect these toxic CDOs, tricking gullible investors into buying the "insured" CDOs, increasing leverage exponentially through the use of derivatives and shadow banking, and awarding themselves tens of millions in bonuses based on short-term performance, etc., which has made finance a pseudo discipline. Studying such phenomena requires expertise in *game theory*, and not estimation of scalable power laws. Since *The Black Swan* does not consider incentives problems and regulatory issues that cause crises such as the credit crisis of 2007–2009, it does not have any prescriptive value for investors or regulators.

5 Conclusion

This paper critiqued the normative theory of investment decisions presented in *The Black Swan* by Taleb (2007, 2010). I considered the implications of the five core principles of the normative black swan theory for investment decision-making over the intermediate to long run. I demonstrate that the portfolio strategy based on the normative black swan theory is myopic and bleeds profusely over time in realistic option markets that exhibit high implied volatilities. Furthermore, the insatiable hunger for positive black swans by the masses often creates financial bubbles/crashes in the intermediate to long run. Also, extreme aversion to negative black swans (e.g., the "medium-risk" corporate bond and stock markets) leads to significant shortfalls in wealth creation for individuals and value destruction for corporations, causing economic depressions. I also correct many fallacies in *The Black Swan* about the Black–Scholes–Merton option pricing model, the modern portfolio theory, the capital asset pricing model, and the stochastic discount factor.

I conclude this paper by concurring with Taleb that the Finance discipline may have become more "pseudo," *but not because of the "white swan" type mathematical modeling techniques used in this field*. The financial sector went through years of excessive deregulation and increasing industry concentration, which combined with the rapid pace of financial innovation altered the incentives and penalties for many players including bank CEOs, bank shareholders, homeowners, bank debtors, and bank depositors. Understanding the concepts of incentives (i.e., myopic or long-term), rules of the games (e.g., influencing the regulations by excessive lobbying), information asymmetry (e.g., manipulation of information), coalition formations (e.g., increased concentration in the financial industry), etc., are more relevant than estimating the power law coefficient of the probability distribution for taming the financial black swans.

Though crafty manipulation became extreme in the years leading up to the recent financial crisis, the transformation of sell-side financial firms from traditional investment banking to "trading houses" that encouraged such manipulation, has been going for more than three decades. Though traditionally sell-side firms made most of their revenues as underwriters, security analysts, and investment advisors, the leveraging of US corporations, homeowners, and the US government starting from the 1980s (i.e., through the use of junk bonds, mortgage bonds, and government bonds, respectively), followed by an explosion of interest rate derivatives in the 1990s, and credit derivatives since the dawn of the new millennium allowed these firms to make significant portions of their revenues by creating new securities and excessive use of derivatives. Over time, increased trading profits through shadow banking and dubious securitizations based on information manipulation replaced basic banking. Thus, the explanation of black swans should focus on manipulation of information, and not on the scandal of prediction.

Endnotes

1 Assuming \$0.05 instead of \$0.0538 changes the Black–Scholes historical volatility to 21.60% from 21.73%, so the very slight change adds expositional simplicity without altering the main conclusions of this section.

2 See Footnote 1.

3 For example, smart investors like Warren Buffett know that insurance is one of the most profitable businesses due to high-risk aversion against black swan events, as most of us want to protect ourselves from these events even at very high costs. Mr. Buffett has put his money where his mouth is and has written billions of dollars of put options betting the market indices will not fall sharply in the very long run. While black swan investors bleed waiting for the black swan events to occur, the billions in premiums received by Mr. Buffett grow in value with the magic of long-run compounding!

4 In a perverse way, option purchases by naïve black swan investors may be partly responsible for creating the option volatility smile, as the desire for instant riches from positive black swans has existed since the days of Tulip mania. However, since a black swan investor is betting on the unknown unknowns, such an investor cannot know what is underpriced and what is overpriced

in the option markets. Since sellers of options are aware of the existence of many positive black swan investors who think almost anything is possible (see Principle 3), and who want to protect themselves from negative black swans at all cost (see Principle 5), they are able to charge high prices for deep OTM options, leading to high implied volatilities for these options.

[5] A black swan portfolio invests 85–90% in safest assets and 10–15% in the riskiest assets with positive black swan exposure, using Principle 1 (i.e., barbell strategy) of the black swan theory.

[6] If the payoffs from lottery tickets followed non-scalable distributions (such as Gaussian) as claimed by Taleb (2010), then the probability of winning would be exponentially smaller for each of the above lotteries, and it would be impossible for anyone to win a multi-million dollar lottery by purchasing a $1 ticket.

[7] "The largest jackpot won was a Mega Millions ticket drawn on March 6, 2007. Two tickets holders split $390 million and both winners chose to receive a cash payment of $116 million. The largest Powerball jackpot was split in 2006 by eight Nebraska meat packers who received $365 million. The cash option was worth $177 million, the second largest prize in U.S. history. In 2005 a group called "The Lucky 7" held the single ticket winning the $315 million Mega Millions drawing. Their lump sum cash prize was $175 million." The above quote is paraphrased from: https://en.wikipedia.org/wiki/Mega_Millions#The_Big_Game_Mega_Millions.

[8] Also, many real assets underlying corporate securities can have option like features. A vast "real" options literature exists on these assets.

[9] The prime corporate bond yield average declined from 4.59% in September 1929 to 3.99% in May of 1931, and to 2.94% by June 1938. Bonds returned a little more than 6% during the 1930s. Short-term fixed income securities or bills returned more than 3% over the same time period.

[10] Investment grade AAA bonds issued by corporations did well during the 2007–2009 credit crisis, except for a slight dip for a few weeks after the Lehman Bankruptcy in September 2008. The investment grade bond market recovered very quickly and produced high positive returns over each of the past three years since then. Only the mortgage CDO-related AAA bonds performed poorly, but the yields on these bonds were significantly higher than the yields on comparable corporate AAA bonds even before the credit crisis hit suggesting that smart investors knew the dubious nature of these bonds.

[11] The second fundamental theorem of asset pricing extends the first theorem by guaranteeing a unique risk-neutral measure under complete markets.

[12] See the vast literature on this topic in Footnote 14.

[13] See Heston (1993), Bakshi *et al.* (1997), Bakshi and Madan (2000), Bates (1996, 2000, 2006), Duffie *et al.* (2000), Pan (2002), and others for theoretical models. See Bates (2000, 2006), Chacko and Viceira (2003), Chernov and Ghysels (2000), Eraker (2004), Eraker *et al.* (2003), Jiang and Oomen (2007), and Pan (2002) for empirical models. Unlike the above models that allow only finite number of jumps over a finite interval, Madan and Seneta (1990) and Carr *et al.* (2002) introduced Levy models that allows an infinite number of "jumps" over a finite interval.

[14] Not all is lost however. Scholes can very accurately predict that it is optimal for both players to randomly play HEADS 28.57% of the time, and TAILS 71.43% of the time. Following this strategy Dilbert minimizes his expected loss to about $0.07143 in each play, or approximately $71.43 total after the game has been played a thousand times. Note that *if* both players played HEADS or TAILS randomly with equal odds, then Dilbert could expect to win, but Catbert knows better and plays HEADS only 28.57% of the time to maximize his gain, conditional on the minimal loss to Dilbert.

[15] See Footnote 15 for the "mixed strategy" solution discovered by Von Neumann (1928).

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