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We were honored to have Nobel Laureates and investment management practitioners share their thoughts on the contributions and recollections of Harry M. Markowitz.

Robert Merton
Massachusetts Institute of Technology (MIT)

I first met Harry Markowitz in 1973 at a Q Group conference in Vale. He suggested we take a walk and talk, which turned into quite the endeavor at 8,200 feet above sea level. Harry, much older and surprisingly spry, bounded up the hill, discussing finance vigorously. Unfortunately, I was so overwhelmed by the pace that I don't recall the substantive details of our conversation.

Unlike Bill and Myron, my direct interactions with Harry were limited after that meeting, making it challenging for me to share personal recollections as they did. Given the profound impact Harry has had on the field of finance, which affects everyone but especially those in this room, I will instead briefly reflect on how his work has influenced our profession over the past 70 years.

We understand that a well-functioning financial system is vital for sustainable economic growth, development, and stability. Financial innovation drives improvements of the financial system. Advances in finance science and in technology, combined with clear economic need as a catalyst for implementation, drive financial innovation. Without a perceived need, even the greatest ideas and technologies can remain unutilized. Thus, understanding and addressing those needs is crucial. The claim of a dichotomy between the real economy and financial economy is pure fiction.

That finance qualifies as a science is not simply because a dozen Nobel laureates in economic sciences including Harry, have been so recognized for their contributions to finance. Finance today holds a collection of coherent methodologies, principles, models, and evolving hypotheses with an avalanche of data to not only test these hypotheses in a systematic rigorous fashion but also to generate new ones. That makes finance a science.

It was not always this way. I mark the onset of the modern era of finance as a science with Harry's pivotal 1952 paper on portfolio selection that brought risk explicitly into the analysis and created a quantitative framework for the trade-off in choice between risk and expected return. Before 1952, investment theories were anecdotal, based on rules of thumb and accounting identities. More importantly, there was almost no data available on stock returns for rigorously testing these theories. Finance was not a science.

In formalizing the concepts of expected return and risk, and importantly, their connection, Harry's work set the stage and functioned as a catalyst for the development of finance as a science. It established the fundamental principle that it is the risk of the investor's portfolio that matters, and not the risk of its individual components. This formal and systematic portfolio-focused approach to investing marked a profound shift in mindset for individuals and institutions, stimulating applied research on its implementation into practice as well as further ground-breaking theoretical research by others on the pricing of capital assets by applying equilibrium conditions to it. As we know, all of this would eventually lead to a complete global transformation of asset management practice.

Risk and its allocation to the best bearers is what much of the financial system is engaged in. If you doubt this, consider a thought experiment: imagine if we could abstract away from uncertainty, even though we know it is there. What would the financial system look like? No need for most financial contracts including insurance or high-value advice services. T-bills would be all you need. Now look at the world's financial system, and you will see first-order what finance really involves: It is overwhelmingly about measuring and managing risk.

Even when estimating expected returns, we do so by using risk measures. Do we simply take the average of IBM's 50-year returns? Absolutely not. All asset pricing models, in one form or another, incorporate risk measures (second moments such as betas and beyond) to determine expected returns. These measures can be estimated from data much more accurately than first-moment expected returns directly. This is crucial because expected returns are notoriously difficult to estimate, requiring an impractical long sample period to achieve statistical significance. Harry was the first to formally elevate risk to the same status of importance as expected return.

The evolution from 1952 onwards had many contributors (including Harry of course) to a remarkable two-decade wave of finance research, both theoretical and empirical, supercharged by the creation of the first large-scale comprehensive database of stock returns at University of Chicago Center for Research in Securities Prices (CRSP). By 1970, finance had become a fully developed science. While there were some specialized pockets of implementation, the science had relatively limited direct impact on mainstream finance practice until the 1970s, when that all changed due in part to a series of major financial and economic shocks, inducing new and significant risks across the financial system, initially in the U.S. and then globally.

For example, we witnessed in 1971 the fall of Bretton Woods, the rules for the international monetary system. Since 1944, no one had to think about currency fluctuations because exchange rates were fixed; suddenly, all currencies were let loose, creating a major new risk.

What other shocks and new risks arose in the 1970s? We saw double-digit inflation, a phenomenon not experienced in the US since the 1920s. Consequently, no one was experienced in dealing with such high levels of inflation, which were accompanied by another rarity, double-digit interest rates (21% at peak in 1981). We also faced the first oil crisis in the early 1970s, with oil prices soaring from about $2.40 a barrel to around $13, and then a second one in the late 1970s, significantly jolting the economy.

The U.S stock market fell by 50% in real terms between mid-1973 and the end of 1974. Mortgage money was scarce in banks; this was partly due to Regulation Q, which capped bank interest payments at 5%. It was obvious that with the government offering double-digit interest rates with favorable tax treatment, few deposits would flow into banks offering 5%. This led to a significant shortage of residential real estate financing. It was not a matter of being creditworthy; there simply was not any money available.

Furthermore, with high inflation, we also experienced high unemployment, reaching 9% at times in the 1970s and spiking to nearly 11% in early 1982 creating a difficult challenge for traditional policy remedies. Available policy tools to address high inflation tend to exacerbate unemployment, and vice versa, a situation referred to as stagflation.

In response to all these shocks, the 1970s and 1980s were marked by an explosion of financial innovation aimed at managing these new and significant risks, quite the opposite of the reaction to the Great Financial Crisis later in 2008 where the tendency was to pull back on innovation and slow down financial activities.

Consider some of the financial innovations developed during this period that we now take for granted: the first option exchange providing insurance for financial securities; financial futures for hedging currencies, interest rates, and stock price risks. Nasdaq, the first electronic stock exchange, emerged, as well as passive-investing index mutual funds. ERISA mandated corporate pension funds and created 401(k) retirement plans in the US. TIAA became the first institution to adopt global stock diversification into its CREF customer-commingled equities fund. None of these existed in their current form before this period.

We also addressed the mortgage funding challenge mentioned earlier by creating a national mortgage market, establishing a global base for funding U.S. residential real estate. The US has not since had the extraordinary loss of funding for residential housing experienced in the 1970s.

Additionally, we saw the development of high-yield and floating-rate bonds, money market funds (a response to Regulation Q), and debt securitization. The abolition of fixed commissions in 1975 was viewed by many as a turning point transforming the stock market from one dominated by individual traders and brokers to an institutional-driven market.

Of particular note was the creation of the interest rate swap. A simple contract to understand but a challenge to implement on a large scale with efficiency. It eliminated forever the need for banks to forecast interest rates and bear their risk to provide desired banking services to their depositors and borrowers.

These innovations could not have been effectively realized without the advances in finance science–quantitative models, theories, and data– that were developed over the preceding 20 years. We had a coincidence of important developments in finance science and greatly improved technological capabilities of the era (that might seem primitive now but were quite advanced at the time). There was, however, a third coincidence—the major risk shocks of the 1970s—that by creating a recognized need, served as a catalyst for the broad and explosive implementation of finance science and technology.

Later adopted around the world, these extraordinary financial innovations that spanned the 1970s-1980s have paid social dividends for five decades since the crisis events that spawned them. It was during that period that finance science and finance practice became inexorably linked. Science drove the practice and practice drove the science. This partnership between finance science and practice has continued to produce remarkable value-enhancing innovations in finance globally to this day.

So, those who view the financial system as separated from the real economy system, as if Wall Street and Main Street are unconnected, are holding onto a fictitious narrative. This dichotomy view is a relic of outdated macroeconomic textbooks.

On this special occasion, I want to remind our professional community that its work matters not just to them or those who study this field, but to the world at large. Harry Markowitz spanned this 70-year transformative period, from the beginning of finance becoming a science. His profound and ongoing impact is evident. His 1952 paper was a catalyst, and it remains one of the cornerstones of finance.

Consider all the progress, all the mathematics and data we have seen. We are still discussing the same fundamental concepts Harry introduced: the interaction between risk and return. Even with the vast improvements, these themes remain central today.

When I discuss these topics on the road or in classrooms, I often share a longer version of this story, highlighting the impact of our field on economic growth and societal development. I always emphasize that finance is a young science, parallel in timing to the biotech industry. Harry Markowitz was there at the outset in 1952, just as Watson and Crick were at the beginning of biotech in 1953.

I used to underscore the youthfulness of our field by noting that its pioneer, Harry Markowitz, was still active. Sadly, I can no longer make that claim. Robert K. Merton, the sociologist of science, defined immortality for a scientist as going beyond simply being remembered for a historical contribution. True immortality is achieved when a scientist's work is still actively used, taught, and practiced today, not merely as history.

Imagine teaching Newtonian mechanics to MIT freshmen today. They learn it not as a historical artifact but as a practical tool to use. It is as if Newton had been there and just stepped out of the room.

More than seven decades after Harry brought us the mean-variance risk-return tradeoff and formulized diversification, we still teach it and apply it in mainstream investment classes and investment practice. So, for both students and practitioners, Harry Markowitz lives. He is just around the corner from our class or trading room.

In that spirit, Harry Markowitz belongs to that rare breed of scientists who have achieved such immortality. His work continues to live and thrive within our field. We should all celebrate being a part of what he brought to us and to the world.

William F. Sharpe
STANCO 25 Professor of Finance, Emeritus
Graduate School of Business, Stanford University

I was privileged to know Harry Markowitz for a very long period of time. I first met him in 1957 after completing my M.A. in Economics at UCLA and my military service I took a position at the RAND Corporation in Santa Monica, California. RAND was very supportive of advanced degree study so I pursued my PhD in economics at UCLA while working there.

My initial choice of a topic for my PhD dissertation was transfer pricing for allocating resources within a firm. When I had the initial work completed, my advisor Armen Alchian told me that Jack Hirshliefer, whose work I was extending, was coming to RAND and UCLA and should take over the project. I introduced myself to Jack who asked for a week to read what I had written. Afterwards, he said politely that he thought there just wasn't a dissertation there. I then went to Fred Weston, a finance professor for whom I had worked as a research assistant, and asked him for ideas. He recalled that I had been very keen on work by Harry Markowitz and even presented a summary of it when I took Fred's seminar. Moreover, Harry was about to return to RAND so I should introduce myself to him. In addition, Armen Alchian, my economics mentor at UCLA had suggested to Harry that he get in touch with me. So we met. I was somewhat in awe and found Harry not only brilliant, but very friendly and supportive. We chatted a few times and I soon decided to turn my PhD research to portfolio theory.

There were three sections in my dissertation. In the first I developed and tested a program to solve the special case in Harry's book in which all correlations among security returns were due to sensitivities to a common factor—that is, a single index model. In the second, I worked with a financial advisor friend of Fred Weston to get inputs for the model and see what the set of efficient portfolios might be. Sadly, they all turned out to be woefully concentrated in a few securities. In the third section I used the single index model, assumed that everyone followed Harry's approach to create portfolios, and then found the properties of the resulting market equilibrium. The results were appealing. All efficient portfolios were invested in the market portfolio plus borrowing and/or lending. Equilibrium expected returns were related to sensitivities to market returns and the riskless interest rate, etc. In 1961, I received my PhD and began my academic career at the University of Washington.

Throughout the time I was working on the thesis, Harry read some of my drafts, looked at my programs and provided feedback. For all practical purposes, he was my thesis advisor.

Here is what Harry wrote in 2002 about this time (in an article titled: Efficient Portfolios, Sparse Matrices, and Entities: A Retrospective", published in "Operations Research).

"My last substantial contribution to the early development of portfolio theory was to advise a young colleague at the RAND corporation who was considering writing his dissertation (for UCLA) on portfolio theory. This led to his first publication: Sharpe 1963".

The article he referenced, based on the first part of my dissertation, was: "A Simplified Model for Portfolio Analysis". It analyzed the properties of markets in which security returns were generated by a single index model which I called the "Diagonal Model" (based on the properties of the covariance matrix). Given this assumption, I showed how it might be used normatively and provided an algorithm for finding efficient portfolios when its conditions held.

I submitted that paper in the fall of 1961, after I started teaching, although it wasn't published until 1963. As for subsequent research, I was enamored with the results of an equilibrium model that I developed in the third section of my dissertation, titled "A Positive Theory of Security Market Behavior". In that analysis, I also assumed that security correlations were generated only by dependence on a single index. In the dissertation I called the variable which measured the magnitude of such dependence simply "B". As many here know, it was later termed "beta".

In any event, given this assumption I showed that, in equilibrium, securities and portfolios would plot along a straight line in a diagram with expected return on one axis and the measure of sensitivity to the market (B or beta) on the other axis.

I felt that I had pulled a great rabbit out of a hat but that I had more or less put it in by assuming the single index model rather than using a full covariance matrix. I agonized over this until I managed to get the same results without the simplifying assumption. I wrote a paper on it in early 1962 but an initial negative review and changes in editors at the Journal of Finance delayed publication to 1964. The paper was titled "Capital Asset Prices - A Theory of Market Equilibrium Under Conditions of Risk" and the relationship was termed the Capital Asset Pricing Model (or CAPM).

Of course all of this work sprang from Harry's prior portfolio theory. He was indeed a key influence on my career.

Let me now turn to a few of the many things I could write about Harry.

First, his fascination with and contributions to programming languages. When he was at RAND in the 1950's and early 1960's he became fascinated with a broad range of decisions under uncertainty and concluded that what was needed was a programming language specifically designed for what were then called Monte Carlo simulations. No problem—he would create one. Working with Bernie Hausner, he developed and implemented SIMSCRIPT—the world's first simulation programming language, a version of which RAND placed in the public domain

When I was at the University of Washington business school in the 1960's I created a course on "Programming Language Appreciation" in which we studied a few key examples including Fortran, Cobol and Simscript. The students were fascinated with Harry's creation and the many things one might do with it.

In the early 1960's Harry and Herb Karr formed a company—California Analysis Center, Incorporated. It developed software for a new version of the Simscript language and contracted for studies utilizing it. In 1968 CACI went public. At the time Harry had about 47% of the stock. Herb and the Vice President of Finance had the rest, and they fired Harry. I remember Harry telling me that for a week thereafter he wandered the trails in Topanga canyon trying to plan his future thereafter.

Second, a visit I made some years later to the office in New York city where he ran a company that used complex proprietary methods to try to create superior investment returns. He and I talked in the outer office but Harry said I was one of a few people that he could not allow into the conference room because I would be able to figure out their secret approach by looking at the writing on the blackboards.

Third, our times at meetings of the Institute for Quantitative Research in Finance, known then and now as the "Q group", founded in 1966 by Dale Berman "with the intention of exploring the application of Modern Portfolio Theory (MPT) to the investment process".

We were both fellows of the Q group and could attend with no fees. Tradition held that attendees would be seated at tables forming the three sides of a rectangle, with the podium at the top. Generally, seating was first come, first seated. However, the seat in the back row at the right as one faced the podium was reserved for Harry and the one to his left was reserved for me. This enabled us to exchange remarks with each other as the session progressed and to sometimes raise our hands to ask questions of the speaker and/or provide comments on the presentation. On most points we were in agreement but there were notable exceptions. Often we would sometimes discuss a presentation after it concluded—shortly or at length.

A long-standing tradition for the Q meetings was a small dinner out organized by Marty Liebowitz. The usual attendees were Marty, Harry and Barbara Markowitz, and my wife Kathy and I. Upon occasion we were joined by Marty and Ellie Gruber and perhaps a guest speaker.

Fourth – as many of you know, in the last 25 years Harry helped form an investment advisory firm and I helped form a different one. That said, we chose to not discuss them with one another.

To conclude: the question sometimes arises: Was Harry an economist, mathematician or operations researcher? The story is that in Harry's dissertation defense, Milton Friedman said that his work certainly wasn't literature. I think Harry would say he was a mathematician. My answer would be that he was all three.

Harry Markowitz was a pioneer, a key contributor to the field of financial economics and a mentor for me and many others. He is gone but his many profound contributions live on.

Panel discussion: A distinguished panel of investment management practitioners will discuss their enduring memory and interaction experience with Harry Markowitz.

Vineer Bhansali, LongTail Alpha

I'm a physicist like Ron (Kahn). When I first met Ron, my brother used to work at BARRA, and I was introduced to Ron and was told he was a physicist. I was still getting my PhD., and my brother said, "Ron's a physicist, and he does this thing called optimization." I think my brother told me about the covariance matrix. I had no interest at all. I didn't even really care. Then, three or four years later, I ended up in the same field by accident, not due to my own choice but primar- ily because of the 1990s recession. But anyway, what I'll talk about today is just some recollections and then a paper by Harry that was especially influential for me. I'm a practitioner. I came from physics and I'm a physicist. I never took a finance course in my life, and I found in my career three people that I have been very fortunate to work with who came up with a framework or approach that was very simple, extremely useful, and gave me a toolkit for my own work.

The first one when I was an undergraduate, was Richard Feynman, and I learned about Feynman diagrams and path integrals. This whole idea of path integrals is a way to simulate the history of all possible worlds, and is an idea in physics that can take any problem and boil it down, even quantum mechanics and quantum field theory, into a very simple machinery that can be used to solve major problems in theoretical physics. So it's not only conceptually very strong, but it's an extremely powerful way to actually implement theoretical physics ideas.

The second one, of course, was a completely different type of experience, and it was working with Bill Gross at PIMCO, who was my boss for about 15 years, who invented this concept of total return in fixed-income investment management. Again, the total return concept is a brilliant concept, very simple. Once you realize how to use it in fixed income, you just can't do it any other way because that's obviously the most complete way of investing in bonds and maybe other asset classes as well.

The third one, of course, was Harry Markowitz, who had some very simple but path-breaking ideas that are not only conceptually sound, but are also great practical tools.

Let's go to my first slide. I was at a JOIM conference in 2017 here, and the topic that I was speaking about was how to beat the machines before they beat you. So you know, essentially, maybe

6 or 7 years ago anticipating what's happening today. So I'll show you a couple of pictures that you see on the screen (of the Barringer Crater). Two pictures very similar, identical almost, but one is fake and one is real. There were about 70 people in the audience, and Harry was sitting in the back, and the question I had was, "Which is the real one, and which is the fake one?" and let me bring it up here. Okay, so this is exactly what I showed on different pages. And I'm not going to quiz you again, because I think you probably know the answer but one is a real picture, and one is a fake picture.

And I asked which one was real to understand how context drives how humans infer. I'll give you the answer here. The right one is the correct picture; it is the true picture at the Barringer crater out in Arizona. But the funny thing is that when I asked only Harry put his hand up and said, "That's the right one," and of course, other people in the audience probably knew what the right picture was, but the point was really not only did he have an answer, but he actually volunteered and basically put his hand out. He took the risk in an audience while the risk was, you know, maybe not 50-50, but there's a good risk of him being wrong in that inference. The left image is just a mirror image. So my question to Harry was, "How did you know?"

I specialize in the area of tail risk or rare events, events that don't happen very frequently. So there's very little statistical data. What is the forecasting methodology when you don't have much data? If you don't have any data, the only thing you have is a Bayesian problem like these pictures. Put this into a machine, figure out what your priors are, and then update with data as it appears to figure out what can happen in the future, and then position your portfolio accordingly. That's a great area to be in today. Because, really, there's no data so there isn't much competition in this area, and it requires logical thinking without lots of data. So we are still somewhat protected. I reached out to Harry afterward, "Look, I have these three problems on forecasting risk of rare events. Well, can you help me?" He thought about it. He confirmed it later and we got him as an advisor. So now I was another one of his clients. We got to speak about once every two or three weeks for about an hour and a half. With his permission, I taped some of those interviews because by that time he tended to forget things so we could replay the tapes to prepare for the next meeting. And yesterday when I was driving down from Newport Beach, I was listening to one of them, which I don't know if I'll have time to share here. But I would love to play one section of it later on for anyone interested. So we worked for about two years on a lot of problems that led to some of the tools we are using indirectly today.

I'll just flash this picture sitting in his office with his medals. Here's Harry, my memory of him! I absolutely loved his library. It was beautifully organized. Alphabetical, some vintage books that he loved to show off, especially on math, and so on. One book that he had, which is not on this shelf. This is the top shelf, by the way. And I like to say, "Look, my book was on the top shelf. There are all four of my books, but that's simply because it's alphabetical by author name. And it was all really well organized. Andrew's (Ang) book is the first one, and Rob Arnott's book is on there as well. But this is just the A's to B's because it was a big library.

But at the bottom of this bookshelf was a big, thick book that every physicist grew up with. It's called "Gravitation" by Misner, Thorne, and Wheeler. It's about gravitation and it's a required core curriculum class for every physicist, and so I asked Harry if had he read that book, and he said something very interesting. This is probably one of the most important messages that I took away from him. He said "Yes, I read it, but there's a big difference between knowing about something and knowing something". And until then, nobody had actually said these words. So Harry said, "Yes, I have read gravitation. I know about it. But I don't think I know it," and that led to a number of great conversations about science and gravitation and quantum field theory, and so on. I probably ended up having to rack my brain and explain a lot of what I do as much as he had to while he was helping us out. Those were great conversations over these one and a half or two-year periods. One corollary of this, which I don't know if he said this or I just made it up, is that there's a big difference between knowing about Markowitz's work and knowing his work. I got all of his books which we now have in our office. And I have tried to read them carefully; all the original papers because a lot of criticisms that have come over time about his work, he had already anticipated them in the 1950s. If you read them carefully, he already said when he wrote his papers, that these are problems that need to be addressed.

Volume 3 of his series on Risk-Return Analysis that came out last year is a very important book personally to me because I work closely with the plumbing in the financial markets. I'm an implementer. I do stuff that improves clients' portfolios. It's all about decision-making and implementation. This is probably the best book in finance I have personally read recently, and the reason is because it asks the most important questions, how do we know? How do we know what expected returns are, and how do we know what variance is, what risk is? And so on. So it gets really deep. I think I've done about two readings. I can only say that so far I know "about" this book. I don't "know" this book yet. It'll probably take me 50 years to figure out all the stuff that Harry actually discovered while writing this book. But it's a basically a wide-ranging book on philosophy and epistemology, whatever you want to call it. It's a very, very interesting deep book that goes into the real questions we face every day as investors.

Okay, so we spent a lot of time walking. Everyone knows Harry loved to walk. Here is a picture of us having lunch at the Iron Pig not too far away from here, now unfortunately gone. And this will be familiar to a lot of you guys. It's the big barbecue place. So we spent multiple hours going to the Iron Pig having big piles of meat and so on, and I have a few great conversations from my time there with Harry, talking about everything ranging from you know, financial markets as games, which is how ultimately we started modeling a lot of the rare event type of things that I'm interested in. Things like why he did not cook with sharp knives while he was writing Volume 3. The conflict emerging conflict between the US and China, and how you can model it as an agent problem. And then why you should always walk on the right side of Harry Markowitz, which is right here. I don't know if you all know this, but he had an accident when he was a graduate student where a light bulb blew up, apparently, and took out his eye so he could only see one out of one side of his eye, which meant that when he was driving he had to have this old Jeep, which he would park right next to a brand new car, because in this car he could actually look out the rear window and on the other one he couldn't. Just some interesting facts about Harry.

Okay, what I'm going to do is in the next couple of minutes I'll leave you with the discussion of just one of Harry's papers that was extremely influential in at least in the way I was able to position our portfolios—and I'll just summarize it. I highly recommend downloading this paper and reading it. It's very simple. It takes the 3 or 4 major principles of the capital asset pricing model and drops the fourth one, which is the ability for everyone to get unlimited leverage, and then works out the consequences. The simple model has 3 securities (see figure). Security 1 is on the X-axis, security 2 is on the Y-axis and security 3 is on the Z-axis which sticks out of the paper. The expected returns and volatility are in the table. Security 1 and security 2 are identical, and security 3 actually has the highest return, highest volatility, but actually has the lowest risk-adjusted return. So it's the most suboptimal security from the risk return perspective. Why is this paper interesting? Because just by doing simple, and this simplicity is a hallmark of all great research, in my view, just by doing simple algebra and arithmetic, you can actually derive some incredible conclusions from this.

So the line that's diagonal LL Prime, that line is basically your efficient frontier starting at C, and in this world there are 2 types of investors. There are investors who can lever as much as they want, and they can short. And then there are investors who cannot leverage who are forced to stay inside of the triangle. So basically, there are 2 types of investors, and a few very interesting consequences follow if there's partial availability of leverage.

I'll give you 3 examples in real life, including the one I'll make a forecast today, or what I believe is going to happen as a consequence of this picture. When leverage is not available to everybody, but is available to some, the people who can lever, they move out to this point out which is P. It is optimal because it's on the optimal frontier line. There are other investors who want to get those higher returns, but they can't get those higher returns without leverage. And assume they cannot lever. So they have to stay inside of the triangle. So what ends up happening is they end in the edge case at point zero-zero where they are holding the highest returning security, obviously, but which has the worst risk to reward ratio and the whole system starts tending in a direction where the presence of leverage from some for some investors drives other investors who cannot explicitly lever into implicit securities that provide them the leverage. Just so they can match the returns of the levered investors. So example one. When I first wrote this paper, this was in 2006. I was at PIMCO, and this was the age of asset-backed securities, and so on. Some investors could lever up by doing synthetic derivatives, CDOs and so on. They moved out to that point P. And they generated high return. Those who couldn't lever ended up having to create securities, or they had need for securities that would give them levered returns without showing leverage. Wall Street obliged and created securities which had packaged implicit leverage inside of it, moving them on that suboptimal frontier. So, as a consequence, everybody held highly levered portfolios that were not market optimal and they were extremely vulnerable to shocks, and we know what happened in 2008. Second example of this was right in 2017 when Larry Harris and I met at a Q group conference, and he asked me about the big, short volatility trade at that time. And my hypothesis was that this leverage driven dynamic was happening again. So again, those who could create explicit leverage, we're doing it through structures and securities, and so on, those who couldn't, were forced into ETFs and ETNs like the XIV, and so on, where they effectively leveraged it up using packaged products, which, as we all know, blew up in 2018.

We are back there now, so the thought I'm going to leave you with is that we are in a world where, as we all know and read about, 28% of the S&P 500 is 5 or 6 stocks. And Nvidia. For good reasons. It's a good company. However, it's a company that is 5 to 6% of the S&P 500 and a retail favorite. Every time a retail option trader buys an option on the stock or the S&P 500 index they are using explicit leverage. You have heard about the explosion in volume in the zero day to expiry options market. Or levered single stock ETFs. They force market makers and delta hedgers to buy the stock to hedge themselves at a higher price. By doing this the price goes up. Then anybody who's allocating to the index via a passive fund with new cash, like an index fund which has to buy a large amount of this implicitly levered up stock has to buy it at a higher price. So somebody's buying it because they can use leverage and they can indirectly force everybody else to buy it, because they're passive, and they have to stay inside the triangle. So it can lead to the situation where index is not actually the market portfolio. It's actually a highly levered off the efficient frontier portfolio which has been boosted up because leverage is available to some exclusively and not available to everybody else. But this can also go into reverse very quickly.

So I'll stop with that I don't think it's going to end well. I believe, like 2006, like 2017. You're probably going to get a sharp reversal in the next 6 months to a year, and this time it may be led by probably passive rebalancing in reverse.

Anyway, I wanted to thank Gifford and JOIM for inviting me to this panel, and am grateful to Harry Markowitz for all that he has given our field. Simple, elegant and practical tools that make sense.

Kenneth Blay, Invesco

I was asked to share some thoughts about my time working with Harry Markowitz. I'll begin by providing some background about myself and the financial services industry and then share some details about Harry's work, his impact on Finance and the industry, and my experience in working with him.

I started my career in what you might call the dregs of the financial services industry as a stockbroker. One of the interesting things I experienced as a function of that career choice was how late mean-variance analysis actually came on the scene. When we think about Harry and mean-variance portfolio optimization, we tend to think that it's been around since the 1950's. But, in fact, it has taken quite some time to become broadly available. In 1972, it came into the institutional world as a function of ERISA (Employment Retirement Security Act) and its guidance around diversification. It took significantly longer for individual investors to begin to benefit from Harry ideas.

I became a stockbroker for PaineWebber in 1996. At the time, financial advice amounted to pitching stocks to investors. I was taught to select 10 stocks, get my stories straight about those stocks, cold call people, share those stories, and get them to buy those stocks. That was financial advice—think about that.

Sometime around 1998, PaineWebber rolled out an advisory platform that offered asset allocation model portfolios. These big firms, like PaineWebber and Merrill Lynch, still hadn't fully embraced this new approach to selling. After all, transactions were a lot more lucrative. You could also generate more transactions by calling clients and getting them to switch from an "overvalued' stock to one that offered better prospects—you generated trades, and you generated revenues. From the firm's perspective, they could whip their financial advisors into generating more business by just getting them to churn more stocks. You didn't get to do that with the asset allocation model portfolios.

Not only that, when I started in the industry, there was a high bar in terms of the commissions you were required to bring in. If you didn't bring in those commissions, you would get fired. It was that simple. So, even if I wanted to adopt the new asset allocation advisory approach, I had to balance between making sure I met my commission goals through high commission trades and only then could I direct assets into the advisory program that paid somewhere between 1 and 1.5 percent on assets under management annually.

Exhibit 1

The "Great Confusion"
Revisiting portfolio theory

Risk-Return Analysis

The objective of the four-volume book was to revisit Part IV: Rational Choice Under Uncertainty of Markowitz (1959)

Volume 1: Revisits chapter X: The Expected Utility Maxim
- The Expected Utility (EU) Maxim
- Mean-variance approximations to EU and Geometric return
- Alternative measures of risk
- The likelihood of various return distributions

Volume 2: Revisits chapter XI: Utility Analysis Over Time
- The portfolio selection context, dynamic systems/programming
- Judgement and approximation
- The Future

Volume 3: Revisits chapter XII: Probability Beliefs
Discussed rational decision-making under uncertainty. What was originally presented in 17 pages now spans a full 296+ page volume that provides a thorough overview of the philosophical and theoretical lineage of Harry's beliefs.

Volume 4: Was to revisit chapter XIII: Applications to Portfolio Theory

The fundamental assumptions of portfolio theory

Rational investing under uncertainty: Rational decision makers / investors may be assumed to follow certain axioms. The expected utility maxim follows from this as does expected utility maximization. Based on von Neumann and Morgenstern (1944), Savage (1954), and Bellman (1957), a rational investor would use subjective probability beliefs when odds are not known to maximize expected utility.

Approximate EU maximization: Markowitz distinguishes between *explicit*, *approximate*, and *implicit* EU maximization. Explicit utility maximization can be a highly complex endeavor. Maximizing a mean-variance approximation to a relevant utility function is the most feasible/practical approach.

Portfolio selection: A risk-averse investor who carefully chooses a portfolio from a mean-variance efficient frontier will approximately maximize expected utility, whether or not they understand or know about the theory of rational behavior, whether or not they understand what expected utility is, and whether or not portfolio distributions are normal. This is what is referred to as *implicit* EU maximization.

An early version of Markowitz addressing the "Great Confusion" can be found in a 2012 article titled *The "Great Confusion" concerning MPT available in pdf form online.*

In 2003, I ended my time as a financial advisor and went to work for 1st Global, a broker/dealer and registered investment advisor (RIA) in Dallas, Texas, where I was tasked with advising the firm's financial advisors on a broad range of issues, including portfolio construction. The advisors there were Certified Public Accountants (CPAs) who we're looking to incorporate financial services into their practice. That experience provided an interesting perspective on the evolution of the industry. It also gave me the opportunity to gain practical experience using an optimizer, developing model portfolios, and managing those portfolios on a discretionary basis.

One of the great things about today's conference is that many of the people that were an important part of driving the transition to asset allocation being a central element of financial advice are here in this room. These are the people whose work I relied on in developing and managing portfolios. One of the most useful books I read on asset allocation was written by Roger Gibson. It provided a very thoughtful and thorough overview of how to approach asset allocation from a practitioner's perspective. The first portfolio optimizer that I used was part of a software package called Ibbotson Encorr, developed by Roger Ibbotson and Tom Izdorek. By today's standards, it wasn't a very user-friendly piece of software, but it gave you the answers you needed—it was most certainly intended for more sophisticated investors and not the mass market. In 2006, Ibbotson Encorr was sold to Morningstar and by 2008 its optimization capabilities were integrated into Morningstar Direct, which is one of the main platforms used by serious financial advisors. Asset allocation wasn't broadly offered to or accepted by individual investors until sometime around this period.

In 2008, during the Global Financial Crisis, we started getting calls from our financial advisors concerned about things they were hearing regarding the non-normality of market returns and how mean-variance portfolio optimization doesn't work with non-normal distributions. We were being asked why we would even use mean-variance if this was the case.

As part of determining a response to our advisors' concerns, we began to think about who we could work with to help us more completely understand the issue. After some contemplation (and drawing on my experience as a stockbroker), I decided I would cold-call Harry Markowitz and see if I could bring him on board to help us with this effort. After several phone calls and a visit to San Diego, Harry agreed to help us with countering the notion that mean-variance didn't work with non-normal distributions.

After a couple of weeks of going back and forth discussing how we should approach the work, Harry, on one of our regular calls, explained "*I've been thinking about what you want me to do. You want me to write a paper on mean-variance and non-normal distributions. I've decided that I'm not going to do that.*" We all fell silent. He then went on to explain "*I'm going to write a book... I'm going to write a book in four parts that will address the problem.*" He would later come to call this problem The Great Confusion—which was the confusion between the necessary and sufficient conditions for the use of mean-variance in practice.

And that's when we set off to revisit his 1959 book and, more specifically, the philosophy behind portfolio theory. Harry always said that he put the philosophical aspects of portfolio theory in the back of the book because he didn't think anyone would want to read about it. He had now determined that it was critically important to revisit those ideas—which were the last four chapters in his 1959 book. So, in short, our initial effort to counter The Great Confusion led to the writing of the Risk-Return Analysis: The Theory and Practice of Rational Investing book.

In Volume one we revisited Chapter X: The Expected Utility Maxim and how that logically leads to utility maximization. Here Harry distinguishes between explicit, approximate, and implicit utility maximization. Explicit maximization means that you try to derive and then maximize someone's unique utility function—something that would be incredibly difficult and impractical. Think about trying to get someone to tell you what their utility function is. You'll find out, soon enough, just how difficult a proposition that is—especially if they don't know what a utility function is. He then explains that, instead of attempting to determine an investor's utility preferences, you can use something like a log-wealth utility function, which is concave and implies that an investor is risk averse. You would then use a mean-variance approximation to that utility function and maximize that approximation. This is what he called approximate utility maximization and is what he proposed in 1952 and, more completely, in his 1959 book.

Everything he did in evaluating different measures of risk wasn't about determining which risk measure was best, or which risk measure best captured downside risk, or anything like that. It was about which risk measure provided the best approximation to the selected utility function.

He originally favored semi-variance, but it was computationally much more difficult. Variance ultimately won because it was practically as good as semi-variance and the math was substantially easier. Harry changed his mind about semi-variance, after writing Volume I, and began to favor variance, given that it had done a much better job for approximations across the various studies we conducted. Since 1952, there has been a lot of additional work done on approximations to expected utility.

Now that we've talked about explicit and approximate utility maximization, we can get to implicit utility maximization. So, how does this get done in practice? Harry explains that even if an investor doesn't know about expected utility, what their utility function is, or any of those types of things, they can implicitly maximize utility by selecting a portfolio on the mean-variance efficient frontier. That is implicit utility maximization and how all of this transcends into practice.

So, Volume 1 was a response to all of the doubters, and we provide plenty of reasons why mean-variance actually works with non-normal distributions. We looked at mean-variance approximations to expecting utility, then mean-variance approximations to the geometric mean, and then Harry shared some work he had done with other researchers on the likelihood of various return distributions for stock returns across the world. This last part was the cherry on top of everything. Indeed! While mean-variance can benefit investors even when distributions are not normal, it is useful to know about some of the distributions investors face. There's a paper that Harry published before the release of the book titled The "Great Confusion" Concerning MPT, that provides a concise overview of many of the arguments made in Volume I. (available online at: https://www.ieb.es/wp-content/uploads/2014/07/11.pdf)

Before I move on to discuss Volume 2, Harry would not be happy with me if I didn't share the following table with you (see Exhibit 2) where he explicitly makes the point about mean-variance approximations to expected utility.

Every time he would talk to someone about the Great Confusion, he would refer to this table. This table is in Markowitz 1959 and Markowitz 2013 and shows the range of returns within which mean-variance does well in approximating expected utility. I produced the chart on the right so you can easily see that it does quite a good job for a pretty broad range of returns.

It's funny, Harry was never really mean about anything. However, after discussing and detailing how mean-variance does not require normal distributions he concludes Chapter 2 as follows:

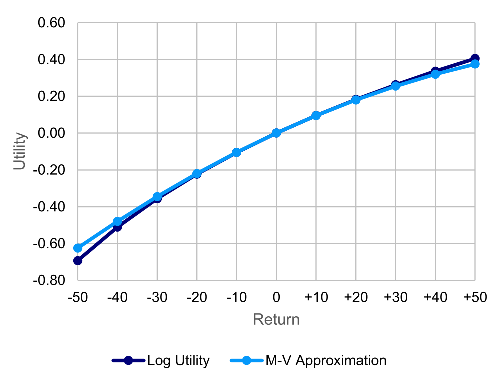
"*It is now over a half-century since Markowitz* (*1959*) *first defended MV* [*mean-variance*] *analysis as a practical way to approximately maximize EU* [*expected utility*]. *In light of repeated confirmation of the efficacy of MV approximations to EU*, *the persistence of the Great Confusion—that MV analysis is applicable in practice only when return distributions are Gaussian or utility functions quadratic—is as if geography textbooks of 1550 still described the world as flat.*"

I didn't know exactly when we realized the world wasn't flat, so I looked it up. It turns out that by 500 BC we already had a model describing the world as round. This was, in Harry's unique way,

Exhibit 2

The Great Confusion
Comparison of log utility and a mean-variance approximation

Return (%) r	Log Utility Ln(1 + r)	Mean-Variance Approximation $r - \frac{1}{2} r^2$
-50	-0.69	-0.63
-40	-0.51	-0.48
-30	-0.36	-0.35
-20	-0.22	-0.22
-10	-0.11	-0.11
0	0.00	0.00
+10	0.10	0.10
+20	0.18	0.18
+30	0.26	0.26
+40	0.34	0.32
+50	0.41	0.38



Source: Markowitz(1959), Markowitz(2013), Invesco.

a pretty sharp rebuttal to those who argued against the efficacy of mean-variance.

We can now move on to Volume 2 where Harry revisits Chapter XI of Markowitz (1959) and covers things like utility analysis over time, multiperiod portfolio selection, and also presents some work we did together on Tax-Cognizant Portfolio Analysis, which was coincidentally published in the JOIM. He also includes a chapter called The Future, which is quite amazing. He spent a lot of time thinking about where things should go. In the first part of this chapter he talks about areas that need to be addressed and researched. In the second part, he talks about how to develop a system that can be implemented to provide portfolio advice in the future. Many of you may not know that he spent a lot of the later part of his life developing what he called his Game of Life simulator, working with GuidedChoice to build out parts of these ideas. He always believed that there was a way to use simulation methods and computers to provide better financial advice to individual investors. So, if you haven't had a chance, you may want to read about what Harry thought about the future and how you might put such a system together. This should be particularly of interest to those people who are building out these systems. He argues about how SAS and SQL code are inefficient and how you might want to do this using what is called an EAS-E (Entities, Attributes, Sets, and Events) framework, which is a reapplication of the SIMSCRIPT programming language he developed.

Volume 3 revisits Chapter XII of Markowitz (1959), which was originally only 17 pages long and explained the philosophy behind portfolio theory. Here he turned that discussion into a roughly 296-page Volume. I've worked through reading this several times. There's plenty of very interesting information and insights in the book but, I will say, it isn't the kind of book where you're going to just sit down, read through it once, and understand everything that was presented. However, it does get you to think about how one should approach things. That was one of the things that Harry always made clear to me. We're not looking at what to do. The way we are approaching this is through understanding how you should act in the face of uncertainty. That's what this is all about. Unfortunately, Harry didn't get to finish Volume 4.

At this point, I should explain why all of this is important. It's important because a lot of what we take for granted today started from Harry's 1952 paper, his 1959 book, and then his work with Bill Sharpe on the one factor model of covariance, which ultimately led to the Capital Asset Pricing Model (CAPM). Those initial ideas were then extended by other leaders in the field to provide the tools and methods that are now central to investment management.

Consider risk models. From the one-factor model of covariance you start getting other risk models developed by Elton and Gruber, Barr Rosenberg, and others. Today, you have entire businesses built on offering risk models. The market portfolio from the CAPM was the beginning of passive investment management and provided the fundamental basis for capitalization weighting. The introduction of the CAPM was also what I call the beginning of the age of portfolio relativity. Everything after the CAPM, from individual securities to diversified portfolios, is considered relative to the market portfolio or a market benchmark. Once we started doing that, we started getting a lot of interesting research about anomalies in security returns that aren't necessarily explained by the CAPM. This led to factors, factor portfolios, multi-factor models of covariance, and performance evaluation methods. We now have the Treynor ratio, the Sharpe ratio, Jensen's alpha, and the information ratio, to name a few. There are so many things that are commonly used today

Exhibit 3

The Age of Portfolio Relativity
Markowitz's ideas transform the theory and practice of investing

Theory

Risk models: The development of Sharpe's 1963 one-factor/market index model of covariance was an early predecessor to modern risk models developed by Elton and Gruber (1973), Rosenberg (1974) and others.

The market portfolio: The development of the CAPM by Treynor (1962), Sharpe (1964), Lintner (1965) and Mossin (1966) introduced the world to the capitalization-weighted "market" portfolio and relates an asset's expected returns to its market risk, or beta.

Factor investing: Reinganum (1981), Fama and French (1992), Carhart (1997) and others identified anomalies in asset prices that countered the efficacy of the CAPM and argued for the inclusion of additional factors, such as size, value and momentum, for more effective asset pricing.

Performance evaluation: Treynor (1965), Sharpe (1966), Jensen (1968), and Treynor and Black (1973) developed some of the most-used performance evaluation metrics in practice today – the Treynor ratio, the Sharpe ratio, Jensen's alpha and the information ratio.

Practice

The shift from selling to advice: As the concept of asset allocation gained traction with financial advisors in the early 2000's, financial advice was transformed from the pitching of individual stocks to the offering of efficiently diversified portfolios customized to address the unique risk/return objectives of individual investors.

Investment management innovations: These include quantitative approaches to active asset management (Grinold and Khan, 1994), improved risk models that allow for better risk targeting and management, and the optimization of excess returns for more efficient use of tracking error (Chow, 1990; Waring et.al., 2000).

Product and customization innovations: As asset allocation and investment management techniques advanced, so did the products and services that are now broadly available to everyday investors. From a broad array of index-based ETFs to factor strategies to the active management of the tax implications of investing, more investment products and services are available today than ever before to facilitate efficient asset allocation and to address specific investor preferences in achieving investor objectives.

in the practice of investment management that are, in one way or another, directly related to Harry's initial ideas.

This takes us back to where we started, when I talked about the shift from pitching stocks to offering investors asset allocated portfolios, or rather the shift from selling to advice. In my estimation, this practice only began to gain traction sometime around 2008 when Morningstar made asset allocation broadly available to financial advisors. This means that the notion of using asset allocation for everyday investors is an innovation that has only really been available, and broadly accepted, for something like 15–20 years. Put another way, we are only getting started in making use of Harry's ideas.

When we think about investment management innovations, we've got Ronald Kahn here with us today who literally wrote the book on active portfolio management. He employed many of Harry's ideas in his work in advancing both active and passive portfolio management methods. In practice, there is a lot of active management that goes into developing and managing passive strategies and products. We have other innovations that increasingly allow for greater customization including the work of Mark Kritzman, Will Kinlaw, and David Turkington who have advanced absolute and relative optimization and risk management methods. These are all innovations that today allow us to provide investment solutions to address a broader set of investor needs and preferences. To me, the next big innovation in asset management will be in being able to create unique client portfolios that incorporate a variety of different aspects of portfolio management, including risk preferences, investment horizons, liability management, ESG preferences, and taxes, and then making those portfolios available in a single client account. This next step for investment management will largely be a function of Harry's initial ideas and all the innovations that followed from those.

Aside from his innovations in portfolio theory, Harry also advanced ideas in many other areas. While I will not be able to go into detail about these today, I do want to point out his work on advancing simulation methods, which was another area where he spent a significant amount of time and effort. This includes his work in developing the SIMSCRIPT programming language, his financial market simulation work with Jacobs and Levy, his efforts around his Game of Life work, and the work we did together in applying

Exhibit 4

Beyond models: Financial market simulation

- **Markowitz (1959) suggests using Monte Carlo methods for specific cases**
- **Developed SIMCRIPT simulation language**
  - An executable simulation modeling language
  - Entities, Attributes, Sets, and Events (EAS-E)
  - Used worldwide by a variety of entities for decision analyses including the U.S. military, the Federal Aviation Administration (FAA), NASA, NATO and over 20 countries, and Lockheed Martin
- **Awarded the John von Neumann Theory Prize by the Institute for Operations Research and Management Sciences in 1989**
- *Investment Rules, Margin, and Market Volatility* (1989)
- *Individual versus Institutional Investing* (1991)
- *Financial Market Simulation* (2004) and JLM Simulator
- *Consumption, Investment and Insurance in the Game of Life* (2015)
- *Tax-Cognizant Portfolio Analysis: A Methodology for Maximizing After-Tax Wealth* (2016)
- *Multiperiod Portfolio Selection: A Practical Simulation-Based Framework* (2020)

simulation techniques for after-tax and multi-period portfolio selection.

I'll close by sharing my personal experience in getting to understand who Harry was as a person through our work and friendship. So, here's a picture of Harry and this younger, better-looking guy working with him (see Exhibit 5). This was the optimization part of working through the portfolio selection process with Harry. It's a day I'll always remember.

Exhibit 5

Harry loved ideas. This was one of the things I absolutely loved about working with Harry. You could share and explore ideas with him. Our contract with Harry was for two one-hour calls per week to advance our work. These one-hour calls often turned into two-, three-, and even four-hour calls. So, I was fortunate enough to spend 8–10 hours a week working with Harry over a period of several years. Of course, half of that time was often spent joking around and listening to Harry sing show tunes. He was such a fun person to work with. I'll say that you might have been stressed or in a bad mood when you started a phone call with him, by the time you finished that call, you would be happy and in a good mood. That's because Harry was such a joyous person—and he transmitted that to people. Everyone loved Harry. . . at least most people did. I think the only person that didn't like Harry was Nassim Taleb. That said, I'm not sure Taleb actually likes anybody.

Harry loved his family. I had the opportunity to have dinner with him at his house a few months before he passed away. I asked him "Harry, looking back, what was the most important thing you accomplished in your life?" He immediately pointed to the wall right over the entrance to his kitchen. It was filled with pictures of his family.

He didn't think twice... his family was the most important thing for him.

Another thing I'll share, and it's something I've heard from other people as well, is that it's hard to understand how he was able to share the amount of time that he spent with everyone. When we worked together, we would talk about things, and he would then assign a task for me to complete. Many of these were non-trivial tasks, some of which I'd never done before. "Ken, we need to put together a model of covariance." "Ken, we need a taxation model to estimate the impact of taxes." "Ken, we need a multi-variate simulator to simulate taxation over time." With each request, I would dutifully go off and figure out how I was going to get Harry what we needed to move things forward.

As I progressed with these efforts, I would generally send my work to him for review in the afternoon. I would often get a response from Harry at around 2 or 3 o'clock in the morning. He said he liked to work and respond to email at that time because it was quiet, and he could focus. He also shared that he did that because he really enjoyed spending time and working with people during the day. That said, I believe this is how COVID ultimately got to him. He never got infected with COVID, but it did isolate him from people. It wasn't until then that I started to perceive him as getting old.

One of the things I shared with him at that last dinner was an old George Burns joke that I would often tell him. I said "*Harry, if you live to be 100, you've got it made. Very few people die past that age.*" He chuckled as he always did when I told him that joke. I then went on to thank him for everything he had taught me and everything he had done for me. I figured that would be the last time I would see him. And it was. I miss him. He was an amazing person and friend and someone who changed my life. I learned a lot from Harry about Finance as well as about being a good person.

Reflecting on his impact on the world of Finance, we are only now beginning to benefit from his ideas. He brought us the idea of portfolio selection in 1952 and where we're going in asset management now is a time when investors are going to have a much broader array of portfolio choices. They'll have greater access to portfolios that are more closely aligned with their unique objectives, needs, and preferences. I believe this is the new era of investing that Harry brought us.

Mark Kritzman
Windham Capital Management

Several years ago, the Financial Analysts Journal asked Harry if he would be willing to be interviewed for publication in the journal, and he agreed on the condition that I be the person to interview him. At least that's what Stephen Brown, who at the time was the FAJ's executive editor, told me. I think Harry wanted me to interview him because he knew that I would ask him about a couple of topics that Harry wanted to be more widely known. So, I'd like to share a story about one of those topics.

This story begins with a course that I have been teaching at MIT for more than 20 years, in which people from industry pose problems for student teams to solve. One such problem was posed by Sebastien Page, who at the time worked for State Street. The problem was to figure out the best strategy for rebalancing a portfolio. On the surface, this problem seems straightforward. A portfolio begins as an optimal blend of assets but as time passes prices change causing the portfolio to drift away from the target optimal weights. At the time, there were two conventional approaches being used to rebalance a portfolio. One approach was calendar based in which the investor would rebalance the portfolio at specified calendar intervals such as a month or a quarter. The other approach was range based in which the investor would rebalance the portfolio when the weights drifted outside of a specified range.

It turns out that the student team that was assigned to address this project was composed of Ph.D. students who studied at Lincoln Labs, a branch of MIT that works on national security issues, and in particular, missile technology. These students were rocket scientists, literally. Although they were well versed in math and physics, they did not know much about finance. So, I taught them about trading costs, utility, and certainty equivalents. And I suggested that they come up with a way of balancing trading costs with optimality. Being rocket scientists, they soon recognized that they could use an algorithm that redirected missiles that had veered off course to redirect a portfolio that had veered off course. I won't go into the details of their solution, because it's a bit complicated, other than to say that it is based on dynamic programming.

In any event, I was so impressed with their solution I encouraged them to rewrite their project paper as a journal article and to submit it for publication, which they did. Their article was published in *The Journal of Portfolio Management*. Soon after it appeared in print, I received a call from Harry. He knew to call me because the students had kindly acknowledged my support. After complimenting their article, he pointed out to me that their solution suffered from the curse of dimensionality, which means that as it is applied to more assets, it becomes computationally impossible to solve. Harry went on to say that he and a co-author, Erik van Dijk, had developed a quadratic heuristic to address the curse of dimensionality. He then asked me to ask the students to test their missile-based solution against his quadratic heuristic. I pointed out to him that the students had graduated and were off doing whatever it is that rocket scientists do. He then asked me to test his quadratic heuristic. So, I enlisted the help of my colleagues, Sebastien Page and Simon Myrgren, and we set out to test Harry and Erik's heuristic. Harry, of course, provided guidance along the way.

It turns out that we were able to show that Harry and Erik's heuristic worked extremely well. It kept pace with the rocket scientists' missile-based solution for up to as many assets as we could test, which was five, and it performed substantially better than calendar-based rules and range-based rules for up to 200 assets.

Harry was thrilled. He asked us to submit our paper documenting these results to an academic operations research journal. I pushed back arguing that we would need to re-write our clear and accessible article by adding superfluous technical jargon and mathematical symbolism, but Harry insisted. So, we re-wrote our article such that it was unapproachable to all but the duly anointed, and we submitted it to Harry's journal of choice.

Our submission was promptly rejected, which was not surprising given that the journal's rejection rate was 90%. Harry then relented and agreed to let us submit our original article to a practitioner finance journal. It was subsequently published in the *Journal of Investment Management*, which made Harry very happy.

Not long after the article appeared in JOIM, I had dinner with Harry at one of his favorite restaurants in La Jolla. During dinner he asked me what reason the operations research journal had given me for rejecting the paper. I told him that the editor acknowledged that Harry and Erik's heuristic worked well in the application we had tested but he was not convinced that it could be generalized. I'll never forget Harry's response, which showed his quick wit. He said, "that's like asking

Newton if his theory works as well with pears as it does with apples."

Harry was very proud of the work we had done to validate his and Erik's quadratic heuristic, and in Part 2 of his three-part opus documenting his life's work, he included a section describing our work. This acknowledgement is a source of great pride for me. I will miss Harry.