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# VOLATILITY MANAGED MULTI-FACTOR PORTFOLIOS

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*This paper demonstrates that portfolio performance can be substantially enhanced by simultaneously utilizing historical factor return volatilities and option-derived market volatilities to optimize factor exposures. The improvements are particularly pronounced in regimes where option-implied market returns exhibit high volatility and right-skewness. Further gains in risk-adjusted portfolio returns are achieved by estimating model parameters separately for different regimes. Qualitatively similar results are obtained when all parameters are estimated strictly out-of-sample. These findings are not limited to a specific set of factors; comparable enhancements are observed when employing principal components derived from a broad set of factors.*

There is broad consensus among academics and practitioners that risk premia in asset markets fluctuate over time. Both theoretical and empirical research in finance has identified a number of variables, such as the business cycle or investor sentiment, that may explain these dynamics. Recently, a literature has emerged that explores volatility measures as conditioning variables to exploit this time-variation in risk-premia and dynamically optimize a portfolio's factor exposure. For instance, Moreira and Muir (2017) document that historical volatility of factor returns offers valuable insights for the management of single-factor portfolios. DeMiguel *et al.* (2024) extend the analysis by considering volatility, estimated from past market returns, as a conditioning variable in the optimization process for multi-factor portfolios. While these studies use volatility estimates based on past returns, Martin (2017) and Martin and Wagner (2019) show both theoretically and empirically, that option-implied market volatilities can be employed to calculate relatively tight lower bounds for assets' expected future risk-premia.

In this paper we focus on portfolio strategies that utilize both past and implied volatility measures as concurrent conditioning variables. We evaluate the performance of these portfolio strategies

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both for the entire sample, but also separately for low and high Chicago Board Options Exchange (CBOE) S&P500 Volatility Index (VIX) regimes, as well as for low and high CBOE option-implied market skewness regimes. Furthermore, we estimate the model parameters separately for these different risk regimes and analyze whether risk regime-dependent parameters affect the performance of volatility management.

Our main findings demonstrate that both volatility measures from past factor returns as well as option-implied market volatility lead to substantial portfolio performance improvements relative to unmanaged benchmarks. However, significant additional performance improvements can be attained when both volatility measures are used jointly as conditioning variables. The fact that the alternative volatility measures do not contain redundant information is clearly apparent when observing their different univariate marginal effects on optimal portfolio factor exposures. Past factor return volatilities are positively related to optimal factor exposures of SMB, CMA and RMW, whereas for MKT, HML and UMD, the marginal effect is negative. The marginal effect of the VIX on optimal factor weights is negative for all factors, except for the market. The fact that an increase in the VIX should be followed by increased exposure to the market factor aligns with (Martin, 2017) who shows that implied market volatility serves as a lower bound for the market risk premium.

Our analysis also documents that the performance effects of volatility management exhibit significant heterogeneity across different risk regimes. It is most effective in regimes where option-implied skewness indicates more right-skewed (less left-skewed) market returns and in regimes where option-implied market volatility is high. In left-skewed or low VIX market regimes, the effects of volatility management are much more moderate. We also estimate the model parameters separately for the different market risk regimes. This leads to a substantial additional improvement in performance. Interestingly, our findings indicate that in more left-skewed market regimes, the exposure to the market factor should be increased in response to an increase in past return volatility. This is contrary to the main result in Moreira and Muir (2017), who emphasize that exposure to the market should be reduced following higher volatilities of past market returns. In our analysis, this observation holds true only in regimes characterized by either right-skewed market returns or elevated VIX levels. Finally, the findings are robust when parameters are estimated strictly out-of-sample and when transaction costs are taken into account. For estimates considering different VIX regimes, the break-even transaction costs are over 160 basis points.

To identify the relation between volatility measures and subsequent factor risk-premia, we analyze multi-factor portfolios with time-varying exposures to individual risk factors, using volatility measures as conditioning variables. Our choice of conditioning variables is motivated by basic insights from asset pricing theory. In the standard setting of a representative mean-variance optimizing investor, the market risk premium is given by $A\sigma_m^2$, where $A$ is the representative investor's coefficient of absolute risk aversion and $\sigma_m^2$ is the variance of end-of-period market returns. Consequently, the risk premium increases proportionally with $\sigma_m^2$, and the Sharpe ratio rises proportionally with $\sigma_m$.

If volatility is subject to clustering, implying that past volatilities predict future volatilities, we should also expect a relation between volatilities of past returns and future risk-premia. There is empirical evidence that past volatilities are indeed related to future risk-premia and Sharpe ratios (see, for example, Moreira and Muir, 2017), but the empirical relation seems to go in the opposite direction: High past factor volatilities correspond to low future risk-premia and Sharpe ratios. One possible explanation for this finding, suggested by Moreira and Muir (2017), is that some investors are slow in responding to increased volatilities. Following a volatility increase, there may therefore be a lag before investors price risky assets accordingly, thereby causing a temporary decrease in the market price of risk.

Finally, the mispricing of risk discussed by Moreira and Muir (2017) may be particularly severe in regimes of high market stress. Almost by definition, during periods of high market stress, as measured by the VIX, more information becomes available, thus increasing the potential for informational frictions. Hence, the increased likelihood of mispricing during heightened market stress may distort the relationship between past return volatilities or risk-neutral option-implied volatilities and risk-premia. Furthermore, Martin (2017) and Martin and Wagner (2019) show that both the market's expected excess returns and the expected excess returns of individual assets should increase with their respective expected risk-neutral second moments. Asset pricing theory also suggests that the market price of risk depends not only on volatility but also on market skewness (Kraus and Litzenberger, 1976; Harvey and Siddique, 2000; Schneider *et al.*, 2020, see). Thus, in general, the impact of volatility changes on risk-premia will depend on return skewness.

The different channels through which volatility and risk-premia are related indicate that past return volatilities and option-implied forward looking volatility measures should contain complementary conditioning information for multi-factor portfolios. It appears advisable to employ them jointly in a multivariate framework. Our empirical approach accounts for this by simultaneously incorporating volatility measures derived from past returns and option-implied volatilities, and estimates the relation between these volatility measures and optimal factor exposures across different skewness and VIX regimes.

Our paper is most closely related to existing work on volatility-managed portfolios. Moreira and Muir (2017) show that past factor volatility, estimated from past daily returns, is a useful conditioning variable to choose time-varying exposure to individual factors, in particular the market factor. Cejnek and Mair (2021) find that volatility management leads to reduced exposures when the most negative cash-flow and discount-rate news are realized. They also conclude that combining the strategy developed in Moreira and Muir (2017) with a strategy based on forward-looking risk neutral variance can be beneficial. Cederburg *et al.* (2020) conclude that the performance benefits of volatility-management cease to persist once trading costs are applied to portfolio implementation. Barroso and Detzel (2021) apply various cost-mitigation strategies to volatility-managed factor portfolios and find that, even under these circumstances, realistic estimates of transaction costs render volatility management unprofitable for all factors, except for the market. DeMiguel *et al.* (2024) focus on volatility management of multi-factor portfolios and find an improved portfolio performance, even if reasonable estimates of transaction costs are considered.[1]

In addition to research on volatility-based factor timing, our work also relates to the broader literature on volatility and expected returns. Several seminal papers have analyzed the relation between assets' volatilities, idiosyncratic volatilities and volatilities due to systematic exposures and risk-premia.[2] Most of these studies find an inverse relationship between volatility measures and risk-premia, the so-called "low-vol anomaly". Our paper relates to this literature by explicitly analyzing dynamic strategies based on volatility measures and shows when factor risks are more or less rewarded. In our multi-factor setting, we get a more differentiated relation between volatilities and risk-premia: when past factor volatilities are high, most factor exposures should be reduced, but for some factors (SMB and CMA) the opposite is the case. We also find that, when considering option-implied market volatility, all factor exposures, except for the market, should be reduced in response to a volatility increase.

The remainder of the paper is structured as follows. First, we outline our empirical strategy and provide details on the data in Section 1. Next, we present results in Section 2, followed by a wide range of robustness checks in Section 3. Section 4 concludes.

## 1 Empirical Strategy and Data

### *1.1 Conditional multi-factor portfolios and volatility measures*

Throughout our analysis, we employ long-short equity factors, where each factor is defined as a zero net investment portfolio. Thus, the dollar-investment in each factor is zero, but a portfolio's exposure to a factor can be scaled up or down arbitrarily.[3] We refer to the vector of factor exposures at time $t$ as "factor weights", $\boldsymbol{w}_t$.

We define the factor weights as the sum of two simultaneously estimated components: an unconditional anchor weight, which is constant over time, and a time-varying component, conditioned on volatility measures. The vector of weights at time $t$, $\boldsymbol{w}_t$ can therefore be written as:

$$\boldsymbol{w_t} = f(\boldsymbol{\Phi}_t; \boldsymbol{\mu}; \boldsymbol{\theta}) \tag{1}$$

$$= \boldsymbol{\mu} + \boldsymbol{\theta}^\top \boldsymbol{\Phi}_t, \tag{2}$$

where $\boldsymbol{\mu}$ represents the vector of passive, non-managed weights for each factor, $\boldsymbol{\theta}$ are coefficients to be estimated representing the sensitivities of each factor weight to the relevant volatility measures, and $\boldsymbol{\Phi}_t$ represents the observed volatility measures at time $t$. We consider both the annualized standard deviation of daily returns for individual factor returns during the preceding month, denoted by $\sigma_{i,t}$, as well as the implied volatility reported by the CBOE for the S&P 500 on the final trading day of the preceding month, denoted as $VIX_t$.

The return of the volatility managed multi-factor portfolio over period $t$ to $t+1$ is then given by

$$r_{p,t+1} = \boldsymbol{w}_t^\top \boldsymbol{r}_{t+1}, \tag{3}$$

where $\boldsymbol{r}_{t+1}$ is the vector of returns of the factor mimicking long-short portfolios over period $t$ to $t+1$. $\boldsymbol{\mu}$ and $\boldsymbol{\theta}$ are estimated simultaneously to maximize the following standard mean-variance objective function

$$\max_{\boldsymbol{\mu},\boldsymbol{\theta}} \quad E[r_{p,t+1}] - \frac{\gamma}{2}\text{var}[r_{p,t+1}], \tag{4}$$

where $\gamma$ is the representative investor's coefficient of absolute risk aversion.[4]

As discussed above, we also estimate the vector of sensitivities, $\boldsymbol{\theta}$, separately for left- and right-skewness regimes and for high and low VIX regimes. In the latter case, factor weights are defined as

$$\boldsymbol{w}_{VIX,t} = \boldsymbol{\mu} + \boldsymbol{I}_v \boldsymbol{\theta}_{Low}^\top \boldsymbol{\Phi}_t + (1 - \boldsymbol{I}_v)\boldsymbol{\theta}_{High}^\top \boldsymbol{\Phi}_t, \tag{5}$$

where $\boldsymbol{I}_v$ represents an indicator function that equals 1 if the VIX measure indicates a low-VIX regime at time $t$ and 0 otherwise. To this end, the sample is divided into periods based on whether the option-implied market volatility VIX is below or above the expanding sample median value.

Similarly, we estimate the vector of sensitivities, $\boldsymbol{\theta}$, separately for left and right-skewness regimes.

In this case factor weights are defined as

$$\boldsymbol{w}_{skew,t} = \boldsymbol{\mu} + \boldsymbol{I}_s \boldsymbol{\theta}_{Left}^{\top} \boldsymbol{\Phi}_t + (1 - \boldsymbol{I}_s) \boldsymbol{\theta}_{Right}^{\top} \boldsymbol{\Phi}_t, \tag{6}$$

where $\boldsymbol{I}_s$ is an indicator function that equals 1 if the skewness measure indicates a left-skewness regime, at time $t$ and 0 otherwise. A particular month is categorized into the left-skewness regime (right-skewness regime) if the option implied market skewness is above (below) the expanding sample median value.

### *1.2 Data*

We use data from the Center for Research in Security Prices (CRSP), encompassing all ordinary common shares (sharecode 10 and 11) traded on NYSE, AMEX, and Nasdaq, spanning the period from July 1967 6 to December 2020. Accounting data are obtained from Compustat and lagged by 6 months, as in Fama and French (1993), assuming the information is available to investors six months after the firm's fiscal year end. The risk-free rate is the one-month Treasury bill rate from Kenneth French's website.[5] We use the CBOE S&P500 VIX from Datastream and the CBOE SKEW Index from the CBOE website for the time period January 1990 to December 2020.[6,7]

To keep the analysis closely related to existing work, we consider a set of widely applied factors, namely those of Fama and French (1993, 2015), and Carhart (1997): the excess market return (MKT), market capitalization (SMB), book-to-market ratio (HML), change in total assets (CMA), revenues minus cost of goods sold, interest expense, and selling, general, and administrative expenses divided by the sum of book equity and minority interest (RMW), and total return from month $t$-12 to month $t$-12 (UMD). Following Fama and French (1993) each factor in our universe is formed by sorting stocks into portfolios according to their size (split at the NYSE median) and to their return characteristic. Factor-mimicking portfolios relying on balance sheet items are held constant for 1 year. While limiting the analysis to the factors defined above may limit and potentially underestimates the benefits of volatility management, it does not address possible implementation challenges that could arise from the definition of these factors. Explicitly accounting for costs of constructing factor portfolios may suggest alternative factor definitions, which can be constructed more efficiently. In our empirical analysis we abstract from these considerations.[8]

The set of volatility measures consists of each factor's standard deviation estimated from the past month's daily returns, $\sigma_{i,t}$, and the expected risk-neutral standard deviation of the market, given by the CBOE S&P500 VIX index at the beginning of each month, $VIX_t$.[9]

## 2 Empirical Analysis

This section presents the main empirical results. The risk-aversion parameter in the representative investor's objective function, denoted by $\gamma$, is hereby set equal to 5.[10] It is important to note that a change in the parameter $\gamma$ linearly impacts the standard mean-variance objective function by altering the optimal weights of the portfolios. Consequently, the Sharpe ratio and $t$-statistics remain unaffected by the particular choice of $\gamma$. We first report the findings for the base-case in Table 1, where we do not distinguish between left and right skew or between high and low VIX regimes. At the beginning of each month, the multi-factor portfolios are adjusted according to the observed volatility measures. In the column labeled $\sigma_{i,t}$, only the individual factor return volatilities, derived from the past month's daily returns are used as conditioning variables. In the subsequent column, the conditioning variable is

**Table 1** Volatility managed multi-factor portfolios.

| | $\sigma_{i,t}$ | $VIX_t$ | $\sigma_{i,t}\&VIX_t$ |
|---|---|---|---|
| $\mu_{MKT}$ | 1.330 | 1.007 | 1.235 |
| $\mu_{SMB}$ | −0.096 | 0.615 | 0.322 |
| $\mu_{HML}$ | 1.789 | 1.972 | 2.252 |
| $\mu_{CMA}$ | 1.942 | 2.630 | 2.138 |
| $\mu_{RMW}$ | 2.466 | 3.682 | 3.644 |
| $\mu_{UMD}$ | 1.704 | 1.436 | 1.588 |
| $\theta_{\sigma,MKT}$ | −1.687 | | −2.281 |
| $\theta_{\sigma,SMB}$ | 6.255 | | 4.269 |
| $\theta_{\sigma,HML}$ | −18.330 | | −8.579 |
| $\theta_{\sigma,CMA}$ | 11.302 | | 12.132 |
| $\theta_{\sigma,RMW}$ | −2.763 | | 1.311 |
| $\theta_{\sigma,UMD}$ | −6.082 | | −4.717 |
| $\theta_{VIX,MKT}$ | | 0.283 | 1.591 |
| $\theta_{VIX,SMB}$ | | −0.247 | −1.268 |
| $\theta_{VIX,HML}$ | | −12.678 | −9.251 |
| $\theta_{VIX,CMA}$ | | −2.458 | −4.713 |
| $\theta_{VIX,RMW}$ | | −5.788 | −6.873 |
| $\theta_{VIX,UMD}$ | | −3.918 | −0.819 |
| $\alpha$ | 0.129 | 0.180 | 0.215 |
| | (4.855) | (6.199) | (6.954) |
| $\beta$ | 0.814 | 0.753 | 0.714 |
| | (15.793) | (16.613) | (15.194) |
| FF5 $\alpha$ | 0.195 | 0.240 | 0.272 |
| | (6.815) | (8.048) | (8.635) |
| SR($f$) | 1.247 | 1.247 | 1.247 |
| SR($f^\sigma$) | 1.531 | 1.656 | 1.748 |
| z($\Delta$SR) | 3.186 | 3.954 | 4.477 |

This table shows results for volatility managed multi-factor portfolios. The first panel displays passive, non-managed weights for each individual factor ($\mu$), followed by optimal sensitivities of each factor to the relevant volatility measure ($\theta$), respectively. The last panel shows results from time-series regressions of each volatility managed multi-factor portfolio on the unconditional factor portfolio: $f_t^\sigma = \alpha + \beta f_t + \epsilon_t$ and Fama and French (2015) 5-factor $\alpha$'s. The optimal unconditional mean-variance factor portfolio's weights are 1.195, 0.448, −0.135, 3.247, 2.070, and 0.942, for MKT, SMB, HML, CMA, RMW, and UMD, respectively. T-statistics are adjusted for heteroscedasticity and reported in brackets. Alphas and Sharpe ratios are annualized. z($\Delta$SR) refers to the Jobson and Korkie (1981) test of the null that SR$(f^\sigma)$ − SR$(f)$ = 0. The sample period is July 1967 to December 2020.

defined by the VIX observed on the last trading day of the previous month. The last column presents the results when both past factor volatilities and the VIX are simultaneously used in a multivariate context. Both the passive weights $\mu_i$ and the sensitivities $\theta_i$ are jointly estimated over the entire sample period, spanning from 1967 to 2020.

The top panel (first six rows) displays the passive factor weights, which are then adjusted based on the observed volatility measure. We find that RMW gets the largest static weight across all volatility models, while the size factor SMB receives the smallest weight in all models. When past factor volatilities are used as conditioning variables, the passive weight for SMB's becomes even negative. Although all other $\mu$'s are positive, there are slight variations across the three different volatility specifications.

The subsequent panel provides the sensitivities to the individual factors' return volatilities, followed by the sensitivities to the VIX. The results suggest that utilizing past factor volatilities and the option-implied market volatility as standalone signals results in distinct implications for optimal factor weights. Past factor return volatilities are negatively related to the optimal portfolio weight of factors MKT, HML, RMW, and UMD whereas a positive relationship is identified for SMB and CMA. Therefore, in response to an increase in past factor return volatilities, four factor weights should be decreased, and two should be increased. Table 1 also shows that factor weight sensitivities to past factor volatilities and to the VIX are quite different, as half of them switch sign. Most notably, while the weight to the market risk factor should be decreased after an increase in past market return volatility, it should be increased in response to an increase in option-implied market volatility, which is in accordance with Martin (2017). Table 1 also shows that all factor weights, except the one for the market factor, should be decreased when the VIX increases.

Referring to the last column, which simultaneously incorporates factor volatilities estimated from returns over the past month and the VIX, we see that the signs of $\theta_{\sigma,i}$ and $\theta_{VIX,i}$ remain consistent with the univariate setting, except for RMW, which switches sign with respect to past factor volatility. Importantly, the marginal effect of the VIX is still negative for all factor weights except for the market factor, even in the multivariate model.

The next panel in Table 1 displays the portfolio performance results. We see that volatility management significantly improves the performance for all three volatility models. When regressed on the returns of the unmanaged portfolios, the resulting alphas are all highly significant and range between 12.9% and 21.5%.[11] In a univariate analysis, using the VIX yields larger $t$-statistics than employing past factor volatilities. However, both models are dominated when both volatility measures are utilized as conditioning variables. We also report Fama and French (1993, 2015) 5-factor $\alpha$'s along with their $t$-statistics and show significant performance improvements across all conditional variables. Volatility management additionally results in substantial Sharpe ratio improvements. For example, when using individual factor volatilities, the Sharpe ratio increases from the unmanaged portfolio's 1.247 to 1.531, marking an increase of more than 20%. The largest improvement in the Sharpe ratio is again achieved when both $\sigma_{i,t}$ and VIX are used as conditioning variables. In this case the Sharpe ratio increases by 0.5, which corresponds to a 40% increase. The final row presents the $z$-statistic from the Jobson and Korkie (1981) test, whether the Sharpe ratios of the unmanaged and the managed portfolio are identical, i.e., $\mathrm{SR}(f^{\sigma}) - \mathrm{SR}(f) = 0$. The results strongly indicate statistically significant improvements in Sharpe ratios, with the most favorable outcome observed when both $\sigma_{i,t}$ and VIX are considered jointly, resulting in a $z$-statistic of 4.477.

### 2.1 Volatility management in different risk regimes

#### 2.1.1 Unconditional factor weights

As discussed before, the relation between volatility and factor risk-premia may be influenced by the prevailing risk regime. To investigate whether this is indeed the case, we assess the performance of volatility managed portfolios separately for different risk regimes: for periods in which the option implied market VIX is above or below the overall sample median (high or low VIX) and for periods in which the option implied skewness index is left- or right-skewed.[12] To this end, we refrain from re-estimating the model parameters separately for each risk regime. Instead we use the parameter vectors $\boldsymbol{\mu}$, $\theta_\sigma$, $\theta_{VIX}$ derived for the overall sample, as reported in Table 1. Table 2 shows that volatility managed multi-factor portfolios exhibit statistically significant alphas and increases in Sharpe ratios across all volatility regimes. Nevertheless, the magnitude of the performance improvement attributed to

**Table 2** Performance in different implied volatility and skewness regimes.

| | Low VIX | | | High VIX | | |
|---|---|---|---|---|---|---|
| | $\sigma_{i,t}$ | $VIX_t$ | $\sigma_{i,t}\&VIX_t$ | $\sigma_{i,t}$ | $VIX_t$ | $\sigma_{i,t}\&VIX_t$ |
| $\alpha$ | 0.052 | 0.033 | 0.062 | 0.223 | 0.274 | 0.345 |
| | (2.970) | (1.742) | (3.161) | (3.053) | (4.545) | (4.687) |
| $\beta$ | 1.023 | 0.870 | 0.917 | 0.657 | 0.512 | 0.496 |
| | (29.906) | (29.136) | (29.442) | (7.037) | (8.510) | (6.936) |
| FF5 $\alpha$ | 0.138 | 0.091 | 0.122 | 0.142 | 0.119 | 0.197 |
| | (4.243) | (3.968) | (4.397) | (2.283) | (3.528) | (3.835) |
| SR($f$) | 1.231 | 1.231 | 1.231 | 1.137 | 1.137 | 1.137 |
| SR($f^\sigma$) | 1.418 | 1.349 | 1.488 | 1.483 | 1.817 | 1.892 |
| z($\Delta$SR) | 1.985 | 1.215 | 2.477 | 1.598 | 2.964 | 2.983 |
| | **Left Skewness** | | | **Right Skewness** | | |
| $\alpha$ | 0.094 | 0.108 | 0.161 | 0.210 | 0.226 | 0.278 |
| | (2.938) | (2.465) | (3.519) | (2.952) | (4.229) | (4.060) |
| $\beta$ | 0.973 | 0.745 | 0.762 | 0.615 | 0.519 | 0.510 |
| | (19.001) | (9.456) | (10.296) | (5.789) | (8.600) | (6.337) |
| FF5 $\alpha$ | 0.162 | 0.096 | 0.153 | 0.217 | 0.186 | 0.243 |
| | (4.332) | (3.236) | (4.110) | (3.211) | (4.333) | (3.950) |
| SR($f$) | 0.964 | 0.964 | 0.964 | 1.273 | 1.273 | 1.273 |
| SR($f^\sigma$) | 1.186 | 1.266 | 1.425 | 1.645 | 1.897 | 1.922 |
| z($\Delta$SR) | 1.529 | 1.738 | 2.429 | 1.695 | 2.845 | 2.669 |

In this table, we split the sample with respect to the full sample median of implied volatility and implied skewness, respectively. The left panels show results in months following low (left) implied volatility (skewness). The right panels show results in months following high (right) implied volatility (skewness).

volatility management varies significantly among sub-samples. It is considerably higher when the implied market volatility is high. A comparison between the top-left and top-right panels of the table reveals that the alpha of the multivariate volatility-managed portfolio experiences a more than fivefold increase, surging from 6.2% to 34.5% per annum when transitioning from a low VIX to a high VIX regime.

Furthermore, the $z$-statistic from the Jobson and Korkie (1981) test indicates that the Sharpe ratio increase resulting from volatility management, whether based on VIX alone or both VIX and past factor volatilities, is most significant in the high-VIX regime. Interestingly, this observation does not hold when conditioning solely on past factor volatilities, indicating that this approach appears to be most effective in the low-VIX regime.

Upon dividing the sample into left-skewed and right-skewed regimes, substantial differences emerge between the two subsets. Comparing the results of the left-skew regime with those of the right-skew regime, it becomes evident that the alpha of the VIX-managed portfolio more than doubles, rising from 10.8% to 22.6%. In the case where volatility management is contingent on both VIX and past factor volatility, the alpha rises to 27.8% in the right-skew regime. Furthermore, we see that all models yield highly significant alphas irrespective of the regime, while only the multivariate volatility-managed portfolio exhibits a significant difference in Sharpe ratio in both regimes.[13]

### 2.1.2 *Regime-specific parameters*

As previously mentioned, the findings on the different risk regimes discussed thus far rely on parameter estimates derived from the overall sample outlined in Table 1. Specifically, the parameters denoting the sensitivities of factor exposures to volatility measures are not independently estimated for different risk regimes. To capture possible structural changes in the relation between past volatilities and future risk-premia across different risk regimes, we proceed to re-evaluate the model parameters separately for each volatility and skewness regime, applying Equations (5) and (6), respectively.

Table 3 summarizes the results. To enhance expositional clarity, we exclusively present results for the multivariate scenario, wherein both the VIX and historical factor volatilities serve as conditioning variables.[14] For easier comparison, the first column incorporates results for the base-case, i.e., parameters are not estimated separately for different risk regimes. The next two columns of Table 3 display results where we estimate parameters separately for regimes characterized by the VIX being below or above its expanding median, and the skewness index indicating left- or right-skewed option-implied market skewness,

**Table 3** Volatility managed multi-factor portfolios for different regimes.

| | Base-case | VIX regimes | Skewness regimes |
|---|---|---|---|
| $\mu_{MKT}$ | 1.235 | 0.959 | 1.032 |
| $\mu_{SMB}$ | 0.322 | 0.286 | 0.344 |
| $\mu_{HML}$ | 2.252 | 2.019 | 1.987 |
| $\mu_{CMA}$ | 2.138 | 2.135 | 1.354 |
| $\mu_{RMW}$ | 3.644 | 3.357 | 3.300 |
| $\mu_{UMD}$ | 1.588 | 1.326 | 1.824 |

**Table 3** (*Continued*)

| | Base-case | VIX regimes | Skewness regimes |
|---|---|---|---|
| | | Low | Left |
| $\theta_{\sigma,MKT}$ | −2.281 | 18.435 | 10.211 |
| $\theta_{\sigma,SMB}$ | 4.269 | −14.118 | −4.076 |
| $\theta_{\sigma,HML}$ | −8.579 | −12.247 | −2.908 |
| $\theta_{\sigma,CMA}$ | 12.132 | −42.796 | 13.091 |
| $\theta_{\sigma,RMW}$ | 1.311 | −7.196 | 4.417 |
| $\theta_{\sigma,UMD}$ | −4.717 | −15.057 | −7.423 |
| $\theta_{VIX,MKT}$ | 1.591 | −1.393 | 1.699 |
| $\theta_{VIX,SMB}$ | −1.268 | 5.576 | −2.780 |
| $\theta_{VIX,HML}$ | −9.251 | −8.211 | −9.582 |
| $\theta_{VIX,CMA}$ | −4.713 | 11.583 | −5.858 |
| $\theta_{VIX,RMW}$ | −6.873 | 2.395 | −8.025 |
| $\theta_{VIX,UMD}$ | −0.819 | −3.427 | 0.317 |
| | | High | Right |
| $\theta_{\sigma,MKT}$ | | −5.111 | −10.338 |
| $\theta_{\sigma,SMB}$ | | 4.991 | 21.554 |
| $\theta_{\sigma,HML}$ | | −3.025 | −2.386 |
| $\theta_{\sigma,CMA}$ | | 10.424 | −33.958 |
| $\theta_{\sigma,RMW}$ | | 1.594 | −9.649 |
| $\theta_{\sigma,UMD}$ | | 13.673 | 2.646 |
| $\theta_{VIX,MKT}$ | | 0.789 | −1.181 |
| $\theta_{VIX,SMB}$ | | −1.528 | 3.126 |
| $\theta_{VIX,HML}$ | | −8.507 | −7.697 |
| $\theta_{VIX,CMA}$ | | −4.767 | 25.697 |
| $\theta_{VIX,RMW}$ | | −6.730 | 2.221 |
| $\theta_{VIX,UMD}$ | | −1.658 | −7.955 |
| $\alpha$ | 0.215 | 0.260 | 0.273 |
| | (6.954) | (8.177) | (7.726) |
| $\beta$ | 0.714 | 0.668 | 0.655 |
| | (15.194) | (14.372) | (12.005) |
| FF5 $\alpha$ | 0.272 | 0.313 | 0.325 |
| | (8.635) | (9.775) | (9.571) |
| SR($f$) | 1.247 | 1.247 | 1.247 |
| SR($f^{\sigma}$) | 1.748 | 1.869 | 1.905 |
| z($\Delta$SR) | 4.477 | 5.132 | 5.320 |

This table shows results for volatility managed multi-factor portfolios for different regimes. We estimate the vector of weights, $\boldsymbol{\theta}$, separately for high and low VIX regimes. The first column shows the baseline estimates, while the second and third columns display estimates based on the levels of implied volatility and implied skewness, respectively. We split the sample at the median implied volatility and implied skewness, and estimate optimal weights with respect to these thresholds.

respectively. The most important findings are contained in the bottom two panels, which showcase the performance of volatility managed portfolios. They reveal that all versions of volatility management exhibit significant performance improvements by distinguishing between VIX or skewness regimes.

Estimating the model parameters separately for VIX regimes improves the alpha from 0.215 to 0.260, accompanied by an increase in their respective $t$-statistics from 6.9 to 8.1. Comparable performance improvements are observed when considering distinct skewness regimes. Here the alpha increases to 0.273, which represents an improvement of more than 25%.

The separate estimation of model parameters also yields to a notable enhancement in portfolio performance when assessed against the FF5 benchmark. For instance, alphas exhibit improvement from 0.27 to 0.31 when accounting for VIX regimes and further to 0.32 when considering skewness regimes.

Inspecting the upper panels, it is noteworthy to observe substantial variation in the marginal effects of past factor volatilities across different risk regimes. In a high $VIX_t$ regime, all factor volatility exposures, except for UMD, exhibit identical signs as in the base-case, i.e., when there is no distinction between different VIX regimes. However, conditional on $VIX_t$ being low, many parameters switch sign. For instance, the $\theta$ estimates for factor volatilities $\sigma_{i,t}$ of MKT, SMB, CMA, and RMW all undergo sign changes. Additionally, the estimates of $\theta_{VIX}$ also change sign during low VIX periods for MKT, SMB, CMA, and RMW.

Similarly, conditional on being in left skewness regimes, the marginal effects of past factor volatilities switch signs for MKT, and SMB, whereas the exposure to UMD should be increased following an increase in implied volatility. During right-skewness periods, exposures to CMA and RMW should be decreased, while UMD exposure should increase, in response to an increase in individual volatility. $\theta_{VIX}$ estimates maintain identical signs during low implied volatility and right-skewness periods.

Therefore, the key takeaway from these results is that the relation between optimal market exposure and market volatility is not constant across risk regimes. Specifically, Table 3 shows that the exposure to the market should not always be reduced in response to an increase in past market volatilities. Both in low-VIX and in left-skewed regimes, one should in fact increase a portfolio's exposure to the market factor, following an rise in past market return volatilities. This is in contrast to one of the main findings of the literature on volatility timing (see, e.g., Moreira and Muir, 2017). In summary, our analysis reveals that accounting for different VIX or skewness regimes captures substantial variation in the marginal effects of risk parameters on optimal factor weights, resulting in substantial enhancements in performance.

## 3 Robustness Analysis

### *3.1 Out-of-sample results*

The results presented in Section 2 are derived from parameters calculated using the entire sample. We now proceed to results for the case where the parameters are exclusively based on out-of-sample data. We hereby estimate the parameters using an expanding window, wherein only data up to time $t$ is considered, and an initial burn-in period is applied to obtain initial estimates. Table 4 shows the results for the time span from 2000 to 2020, employing both the VIX and past factor volatilities as conditioning variables.[15] Results for $\mu_i$ and $\theta_i$ are presented as averages over the out-of-sample period.

As indicated in Table 4, volatility managed multi-factor portfolios demonstrate notably significant alphas when regressed on a static factor portfolio, even when parameters are estimated using out-of-sample data. The multivariate model $\sigma_{i,t}\&VIX_t$ estimated for different VIX regimes, for example, yields an alpha of 31.8% p.a. with a $t$-statistic of 2.5. Furthermore, when assessed against the FF5 factor model, the volatility-managed portfolio exhibits large alphas, albeit marginally significant. In addition, the Sharpe ratio rises from 0.75 for the unmanaged portfolio to 0.8, though the increase is statistically not significant. When we split the sample according to Skewness regimes, the results are weaker.

Figures A.1 and A.2 in the Supplementary Material show out-of-sample thetas for VIX and skewness regime estimates. Several noteworthy results emerge from our analysis. First, thetas exhibit more time variation during the early sample period, as they are estimated less precisely. Second, theta estimates for low-VIX and right-skewed market regimes display larger absolute values, compared to thetas for high-VIX and left-skewed market regimes. This accords with the earlier findings, that the marginal contribution from volatility management is particularly significant during these regimes. Third, it is notable that all three NBER recession dates (highlighted by shaded areas) in our sample coincide with high VIX regimes. In contrast, skewness regimes exhibit a different relation to the business cycle: at the beginning of recessions, returns are more right-skewed, whereas at the end of the recession, returns exhibit more left-skewness.[16] These findings suggest that volatility management should work particularly well during the onset of recessions, as they are characterized by high VIX and left-skewed implied future returns. Fourth, our sample period exhibits an upward trending CBOE Skewness Index (see Figure 1), i.e., implied returns are becoming more left-skewed towards the end of the sample. Consequently, the expanding median skewness increases gradually, and no right-skewed conditions are observed beyond 2009.

Generally, the out-of-sample estimation leads to larger variation in the $\theta_i$ coefficients across factors, consequently prompting more pronounced portfolio adjustments in response to changes in volatility. This may lead to more volatile returns, aligning with the observation that, while alphas of the volatility managed portfolio increase, the impact on Sharpe ratios remains ambiguous. When interpreting the time-variation of $\theta_i$ coefficients both in Tables 3 and 4, it is essential to recall that transaction costs have considerably decreased over this period. This could have resulted in changing risk-premia for the different factors, as they are associated with different trading requirements. For example, maintaining exposure to the momentum factor necessitates a much larger portfolio turnover compared to an equivalent exposure to the size factor. This may also imply time-varying $\theta_i$ in our setting, since we do not explicitly incorporate transaction costs. We therefore explore the effects of such frictions next.

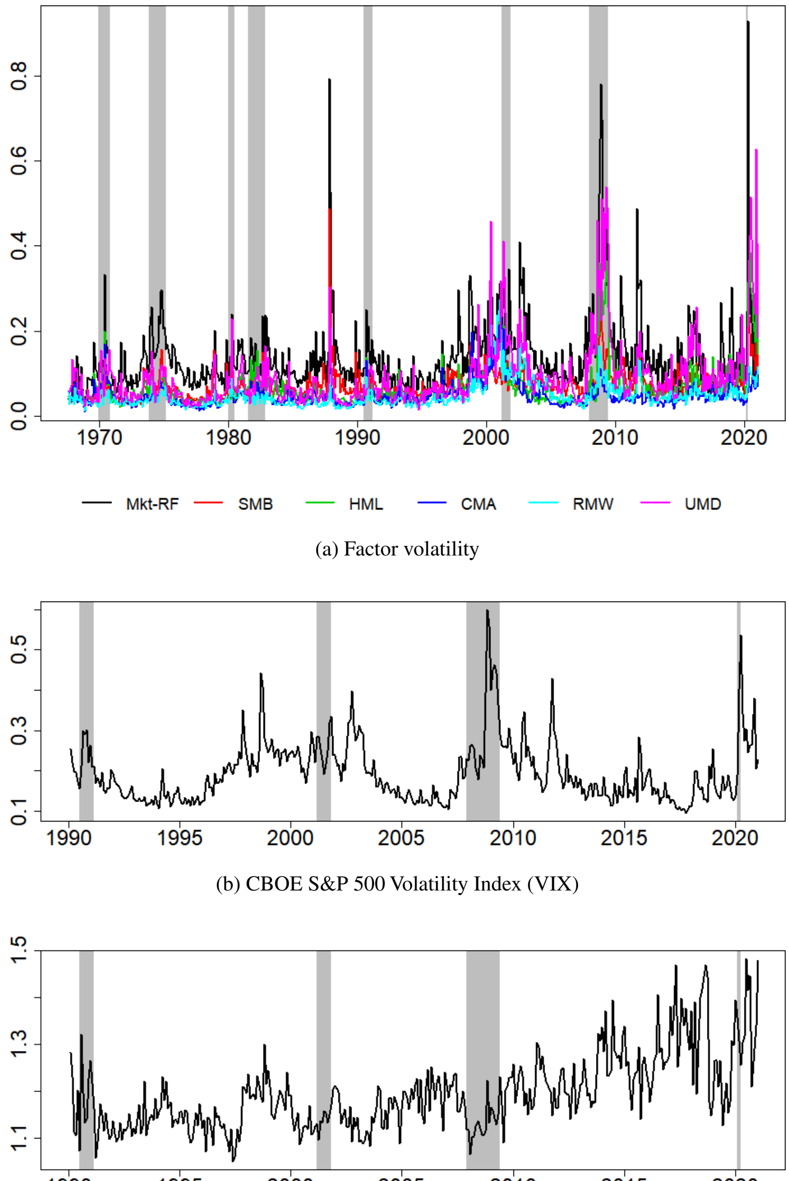
### *3.2 Transaction costs*

To asses the impact of transaction costs on the performance of volatility managed multi-factor portfolios, we calculate the moving average bid-ask spread, defined as (ask price – bid price)/ask price, over the most recent quarter.[17] Figure 2(a), first displays some descriptive statistics for the value-weighted round-trip transaction costs for small and large stocks over time and illustrates the dramatic drop in transaction costs over time, particularly for smaller firms, from 1990 to 2010. Transaction costs for large-cap stocks also experience a decline, especially during the period from the early years of the millennium to 2010, though the decrease is less pronounced than that for small-cap stocks.

**Table 4** Volatility managed multi-factor portfolios for different regimes out-of-sample.

| | In-sample estimates | | Out-of-sample estimates | |
|---|---|---|---|---|
| | VIX regimes | Skewness regimes | VIX regimes | Skewness regimes |
| $\mu_{MKT}$ | 0.960 | 1.034 | 0.535 | 0.429 |
| $\mu_{SMB}$ | 0.286 | 0.344 | 0.122 | 0.032 |
| $\mu_{HML}$ | 2.022 | 1.990 | 1.320 | 1.118 |
| $\mu_{CMA}$ | 2.138 | 1.356 | 1.235 | 0.669 |
| $\mu_{RMW}$ | 3.363 | 3.306 | 2.328 | 1.755 |
| $\mu_{UMD}$ | 1.328 | 1.827 | 1.160 | 1.134 |
| | Low | Left | Low | Left |
| $\theta_{\sigma,MKT}$ | 18.435 | 10.227 | 8.436 | 2.647 |
| $\theta_{\sigma,SMB}$ | −14.118 | −4.082 | −20.530 | −1.561 |
| $\theta_{\sigma,HML}$ | −12.247 | −2.913 | −0.010 | −1.462 |
| $\theta_{\sigma,CMA}$ | −42.796 | 13.112 | −30.265 | 3.576 |
| $\theta_{\sigma,RMW}$ | −7.196 | 4.424 | −27.260 | 1.446 |
| $\theta_{\sigma,UMD}$ | −15.057 | −7.435 | −20.970 | −3.727 |
| $\theta_{VIX,MKT}$ | −1.393 | 1.702 | −0.136 | 0.630 |
| $\theta_{VIX,SMB}$ | 5.576 | −2.785 | 4.895 | −1.321 |
| $\theta_{VIX,HML}$ | −8.211 | −9.597 | −6.719 | −4.153 |
| $\theta_{VIX,CMA}$ | 11.583 | −5.868 | 13.192 | −2.713 |
| $\theta_{VIX,RMW}$ | 2.395 | −8.038 | −1.346 | −2.803 |
| $\theta_{VIX,UMD}$ | −3.427 | 0.317 | −5.807 | 0.627 |
| | High | Right | High | Right |
| $\theta_{\sigma,MKT}$ | −5.119 | −10.354 | −1.254 | −1.450 |
| $\theta_{\sigma,SMB}$ | 4.999 | 21.588 | 12.513 | 11.368 |
| $\theta_{\sigma,HML}$ | −3.029 | −2.390 | −2.454 | −2.558 |
| $\theta_{\sigma,CMA}$ | 10.440 | −34.011 | 10.678 | −8.614 |
| $\theta_{\sigma,RMW}$ | 1.596 | −9.664 | 11.166 | −3.554 |
| $\theta_{\sigma,UMD}$ | 13.694 | 2.650 | 10.076 | 0.107 |
| $\theta_{VIX,MKT}$ | 0.790 | −1.183 | 0.841 | −0.098 |
| $\theta_{VIX,SMB}$ | −1.530 | 3.130 | −1.061 | 4.108 |
| $\theta_{VIX,HML}$ | −8.521 | −7.709 | −4.201 | −7.356 |
| $\theta_{VIX,CMA}$ | −4.775 | 25.737 | −4.223 | 14.955 |
| $\theta_{VIX,RMW}$ | −6.741 | 2.225 | −4.149 | −1.873 |
| $\theta_{VIX,UMD}$ | −1.661 | −7.967 | −0.097 | −7.440 |
| $\alpha$ | 0.326 | 0.314 | 0.318 | 0.234 |
| | (5.748) | (5.243) | (2.514) | (1.990) |
| $\beta$ | 0.521 | 0.484 | 0.014 | −0.022 |
| | (7.890) | (6.083) | (0.091) | (−0.128) |
| FF5 $\alpha$ | 0.262 | 0.251 | 0.242 | 0.181 |
| | (5.298) | (4.978) | (1.785) | (1.447) |
| SR($f$) | 0.934 | 0.934 | 0.752 | 0.752 |
| SR($f^{\sigma}$) | 1.763 | 1.739 | 0.803 | 0.568 |
| z($\Delta$SR) | 3.848 | 3.654 | 0.163 | −0.586 |

This table shows results for volatility managed multi-factor portfolios for different regimes in- and out-of-sample. We split the sample at the (expanding) median implied volatility and implied skewness, and estimate optimal weights with respect to these thresholds. The left part reports the in-sample estimates, the right part shows out-of-sample estimates.




(a) Factor volatility

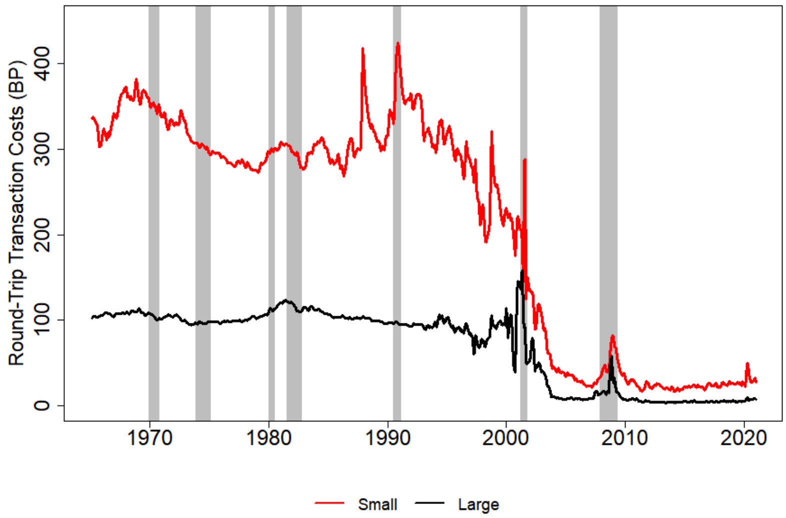
(b) CBOE S&P 500 Volatility Index (VIX)

(c) CBOE SKEW Index

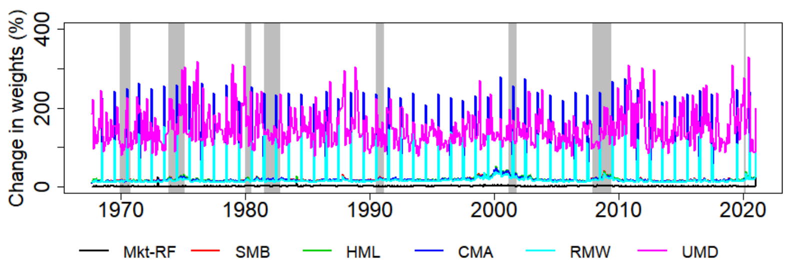
**Figure 1** Factor volatility, implied volatility, and implied skewness.

The figure displays annualized individual factor volatilities, the CBOE VIX index and the CBOE Skewness index. The shaded areas represent NBER recession dates.

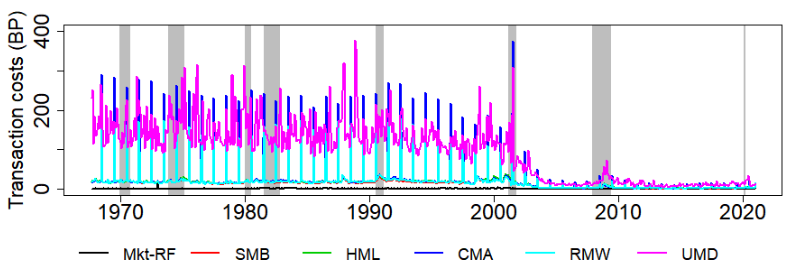
To evaluate the heterogeneous effect of transaction costs on the factors under consideration, we calculate the realized turnover implicit in each factor replicating portfolio as $\Sigma_{s_i}|w_{s_{it}} - w_{s_i,t-1}| \times 100$, i.e., the sum of the absolute values of changes of the weights of stocks (see Figure 2(b)). It indicates that the market factor portfolio is characterized by relatively low turnover, primarily influenced by listings and de-listings. In contrast, the turnovers required for other factors are much larger, with the momentum factor (UMD) exhibiting the largest turnover. In Figure 2(c), we plot




(a) Round trip bid-ask spread




(b) Change in weights




(c) Transaction costs

**Figure 2** Round trip bid-ask spread, change in weights and transaction costs.

Panel (a) displays the value-weighted round-trip transaction costs for small and large firms over time. Small stocks are those below the NYSE median market capitalization. Large stocks are those above the NSYE market capitalization. We use realized transaction costs for the stocks underlying the factors, estimated by the moving average bid-ask spread, (ask price – bid price)/ask price, over the most recent quarter. If in a given month bid or ask quotes are missing, we then substitute the past month's bid-ask spread. If we do not have any bid-ask information for a stock in a given quarter, we assign the median transaction costs of firms in the same size decile. Panel (b) presents turnover associated with the factor portfolios. In Panel (c) we plot the resulting transaction costs. The shaded areas represent NBER recession dates.

**Table 5** Volatility managed multi-factor portfolios for different regimes with TC.

| | No transaction costs | | Transaction costs | |
|---|---|---|---|---|
| | VIX regimes | Skewness regimes | VIX regimes | Skewness regimes |
| $\mu_{MKT}$ | 0.959 | 1.032 | 0.437 | 0.542 |
| $\mu_{SMB}$ | 0.286 | 0.344 | −0.205 | −0.008 |
| $\mu_{HML}$ | 2.019 | 1.987 | 0.711 | 0.889 |
| $\mu_{CMA}$ | 2.135 | 1.354 | −0.009 | −0.027 |
| $\mu_{RMW}$ | 3.357 | 3.300 | −0.028 | 0.061 |
| $\mu_{UMD}$ | 1.326 | 1.824 | 0.000 | 0.000 |
| | Low | Left | Low | Left |
| $\theta_{\sigma,MKT}$ | 18.435 | 10.211 | 2.977 | 4.829 |
| $\theta_{\sigma,SMB}$ | −14.118 | −4.076 | −1.245 | 0.093 |
| $\theta_{\sigma,HML}$ | −12.247 | −2.908 | 0.890 | −3.122 |
| $\theta_{\sigma,CMA}$ | −42.796 | 13.091 | 0.599 | 0.528 |
| $\theta_{\sigma,RMW}$ | −7.196 | 4.417 | 5.106 | 4.128 |
| $\theta_{\sigma,UMD}$ | −15.057 | −7.423 | 1.759 | 0.000 |
| $\theta_{VIX,MKT}$ | −1.393 | 1.699 | −1.009 | 1.324 |
| $\theta_{VIX,SMB}$ | 5.576 | −2.780 | 3.820 | −0.024 |
| $\theta_{VIX,HML}$ | −8.211 | −9.582 | 0.127 | −5.166 |
| $\theta_{VIX,CMA}$ | 11.583 | −5.858 | 0.443 | 0.294 |
| $\theta_{VIX,RMW}$ | 2.395 | −8.025 | 1.781 | 1.830 |
| $\theta_{VIX,UMD}$ | −3.427 | 0.317 | 0.000 | 0.000 |
| | High | Right | High | Right |
| $\theta_{\sigma,MKT}$ | −5.111 | −10.338 | 3.503 | −5.103 |
| $\theta_{\sigma,SMB}$ | 4.991 | 21.554 | 1.915 | 5.959 |
| $\theta_{\sigma,HML}$ | −3.025 | −2.386 | −2.517 | 1.130 |
| $\theta_{\sigma,CMA}$ | 10.424 | −33.958 | −0.044 | −0.232 |
| $\theta_{\sigma,RMW}$ | 1.594 | −9.649 | 5.407 | −1.360 |
| $\theta_{\sigma,UMD}$ | 13.673 | 2.646 | 1.610 | 0.000 |
| $\theta_{VIX,MKT}$ | 0.789 | −1.181 | 0.985 | −1.209 |
| $\theta_{VIX,SMB}$ | −1.528 | 3.126 | −0.234 | 0.131 |
| $\theta_{VIX,HML}$ | −8.507 | −7.697 | −3.833 | −1.322 |
| $\theta_{VIX,CMA}$ | −4.767 | 25.697 | 0.489 | 1.032 |
| $\theta_{VIX,RMW}$ | −6.730 | 2.221 | 2.216 | 3.967 |
| $\theta_{VIX,UMD}$ | −1.658 | −7.955 | 0.000 | 0.000 |
| $\alpha$ | 0.260 | 0.273 | 0.054 | 0.055 |
| | (8.177) | (7.726) | (4.960) | (4.648) |
| $\beta$ | 0.668 | 0.655 | 0.614 | 0.599 |
| | (14.372) | (12.005) | (12.307) | (9.472) |
| FF5 $\alpha$ | 0.313 | 0.325 | 0.037 | 0.036 |
| | (9.775) | (9.571) | (3.296) | (3.056) |
| SR($f$) | 1.247 | 1.247 | 0.478 | 0.478 |
| SR($f^{\sigma}$) | 1.869 | 1.905 | 0.849 | 0.855 |
| z($\Delta$SR) | 5.132 | 5.320 | 3.036 | 3.025 |

This table shows results for volatility managed multi-factor portfolios for different regimes before and after transaction costs. We estimate the vector of weights, $\boldsymbol{\theta}$, separately for high and low VIX regimes. We split the sample at the median implied volatility and implied skewness, and estimate optimal weights with respect to these levels. The left columns report estimates in the absence of transaction costs, the right columns show estimates incorporating transaction costs.

the resulting transaction costs. First, we note that factor portfolios that are infrequently rebalanced (MKT), or rebalanced only once a year (SMB, HML, CMA, RMW) experience periodic spikes in transaction costs over time. In contrast, the momentum factor (UMD) is rebalanced monthly and consistently experiences high trading costs. Second, there is a discernible pattern of elevated trading costs before 2000, followed by lower trading costs during the last two decades of our sample.

Table 5 presents findings incorporating estimated transaction costs. For brevity, we exclusively present outcomes pertaining to scenarios where both factor volatilities and the VIX serve as conditioning variables.[18] To facilitate comparison, results without transaction costs are presented on the left side of Table 5, while the right side showcases outcomes accounting for transaction costs. The enhanced portfolio performance attributable to volatility management remains even when accounting for the impact of trading costs. As expected, however, the alphas and Sharpe ratios of volatility-managed portfolios decrease once transaction costs are taken into account. Compared to the results without transaction costs, the optimization leads to substantially lower unconditional non-managed weights ($\mu_i$) for both VIX and skewness regimes. In addition, most of the $\theta_i$ estimates exhibit lower values, and some undergo a change in sign. This means that factor weights demonstrate a reduced level of responsiveness when transaction costs are incorporated.[19] In addition, we see that unconditional weights are essentially zero for SMB and UMD, and substantially lower for MKT, HML, CMA and RMW. Overall, the outperformance of volatility managed portfolios survives the presence of transaction costs both when evaluated relative to unmanaged multi-factor portfolios, but also when measured against the FF-5 factor model.

Overall, our results differ from those in Barroso and Detzel (2021) who conclude that after transaction costs, volatility management applied to asset-pricing factors, with the exception of the market factor, produces zero abnormal returns. In accordance with DeMiguel *et al.* (2024), our results indicate that for multi-factor portfolios, volatility management can improve portfolio performance even when realistic transaction costs are considered.

Table A.5 in the Supplementary Material provides additional insights into how trading costs impact the profitability of volatility management. Our sensitivity analysis shows that the critical level of transaction costs required to drive the alphas to zero is roughly 150 basis points.

## 4 Conclusion

This paper investigates the role of volatility measures in optimizing dynamic multi-factor portfolio weights in several novel ways. We demonstrate that conditioning factor exposures jointly on volatility estimates from historical factor returns and on forward-looking option-implied market volatilities, leads to robustly improved portfolio performance, exceeding the performance achieved when considering each individual volatility measure separately. The effectiveness of volatility management varies significantly across different risk regimes. Specifically, our results show that dynamically adjusting portfolio weights is considerably more effective during periods with high VIX and right-skewed market returns. These improvements are even more pronounced when we “double sort” based on both VIX and market skewness, with the greatest benefits observed in regimes characterized by both high market VIX and right market skew. Conversely, in a low VIX and left-skewed market regime, the impact of volatility-managed portfolios is much more limited.

Building on these findings, we examine the relationship between volatility measures and subsequent factor risk-premia across different market risk regimes. Estimating the model separately for distinct VIX or skewness regimes leads to substantially increased FF-5 alphas, and Sharpe ratios compared to portfolio obtained when parameters are constant across different VIX or skewness regimes. These results remain robust out-of sample and when accounting for transaction costs. Finally, we show that our findings are not limited to the specific set of factors analyzed in the main analysis. Qualitatively similar results are observed when employing the first five principal components derived from a broad set of 168 factors.

While this paper's analysis abstracts from several practical implementation issues, the findings can provide a useful compass for practitioners. First, the paper provides guidance on how past factor volatilities and forward looking expected market return volatilities should be mapped into factor exposures. Specifically, when past factor volatilities are high, most factor exposures should be reduced, but for SMB and CMA the opposite is the case. The results also suggest that, when considering option-implied market volatility, all factor exposures, except for the market, should be reduced in response to a volatility increase.

Second, the paper can provide guidance for practitioners when deciding in which market environments factor timing has the biggest potential and in which market environments factor timing should be expected to be less valuable. When VIX levels are elevated and implied market returns are right-skewed, the potential benefits from taking more pronounced factor positions are largest. On a more fundamental level, the paper also suggests why dynamic factor strategies can be expected to have more capacity than static strategies: most factor exposures should be dynamically increased in times when volatility is low and, presumably, these are also periods when overall market liquidity is high and transaction costs are low.

**Acknowledgment**

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## Appendix A

### *A.1 Constant parameters: Unconditional sorts*

In Table A.1 we report the results from unconditional double sorts on implied volatility and implied skewness. Despite the unconditional sort, we obtain approximately equal numbers of observations for each of the four buckets. The upper left panel of the table shows results for low implied volatility and left-skewed option-implied market returns. Several interesting results emerge. First, within this category, volatility management of multi-factor portfolios results in the smallest performance improvements. The alphas remain positive; however, they are only statistically significant when past factor volatilities are employed as a conditioning variable. Increases in Sharpe ratios for these regimes do not reach statistical significance at the 5% level.

Next we consider the upper right panel, where implied volatility is low, but the market is more right-skewed. Compared to the left panel, we find substantially larger (statistically significant) alphas accompanied by increases in Sharpe ratios due to volatility management. For example, the multi-factor portfolio managed using the previous month's VIX demonstrates a statistically significant alpha of 21.7%. However, neither the FF5 $\alpha$ nor the corresponding $z$-statistic yield significant results. When both VIX and past factor volatilities are employed as conditioning variables, the FF5

**Table A.1** Performance in different VIX and skewness regimes unconditional sorts.

| | Low VIX, left Skewness | | | Low VIX, right Skewness | | |
|---|---|---|---|---|---|---|
| | $\sigma_{i,t}$ | $VIX_t$ | $\sigma_{i,t}\&VIX_t$ | $\sigma_{i,t}$ | $VIX_t$ | $\sigma_{i,t}\&VIX_t$ |
| $\alpha$ | 0.052 | 0.025 | 0.058 | 0.153 | 0.217 | 0.296 |
| | (2.129) | (1.021) | (2.176) | (2.214) | (2.329) | (3.017) |
| $\beta$ | 0.940 | 0.900 | 0.893 | 0.972 | 0.668 | 0.688 |
| | (18.021) | (16.859) | (14.854) | (14.468) | (6.045) | (6.832) |
| FF5 $\alpha$ | 0.110 | 0.073 | 0.094 | 0.135 | 0.050 | 0.128 |
| | (2.695) | (2.503) | (2.729) | (1.939) | (1.136) | (1.938) |
| SR($f$) | 0.757 | 0.757 | 0.757 | 1.194 | 1.194 | 1.194 |
| SR($f^\sigma$) | 1.013 | 0.856 | 1.052 | 1.443 | 1.693 | 1.853 |
| z($\Delta$SR) | 1.730 | 0.708 | 1.825 | 1.042 | 1.645 | 2.008 |
| | High VIX, left Skewness | | | High VIX, right Skewness | | |
| $\alpha$ | 0.044 | 0.049 | 0.066 | 0.258 | 0.309 | 0.371 |
| | (1.795) | (1.557) | (2.250) | (2.365) | (3.867) | (3.533) |
| $\beta$ | 1.078 | 0.845 | 0.931 | 0.488 | 0.429 | 0.394 |
| | (27.506) | (21.686) | (26.477) | (4.052) | (6.612) | (4.171) |
| FF5 $\alpha$ | 0.106 | 0.083 | 0.109 | 0.173 | 0.148 | 0.217 |
| | (2.169) | (2.314) | (2.573) | (1.713) | (2.672) | (2.485) |
| SR($f$) | 1.749 | 1.749 | 1.749 | 1.104 | 1.104 | 1.104 |
| SR($f^\sigma$) | 1.865 | 1.924 | 1.986 | 1.519 | 1.915 | 1.917 |
| z($\Delta$SR) | 0.913 | 1.250 | 1.693 | 1.263 | 2.476 | 2.231 |

This table shows performance statistics for unconditional sorts of implied volatility and implied skewness. We split the sample with respect to the full sample median of implied volatility and implied skewness. The upper left part, for example, shows results in months following low implied volatility and left implied skewness. The vector of factor weights at time $t$, $\boldsymbol{w}_t$ is given by $\boldsymbol{\mu}$ represents the vector of passive, non-managed weights for each factor, $\boldsymbol{\theta}$ are coefficients to be estimated representing the sensitivities of each factor weight to the relevant volatility measures, and $\boldsymbol{\Phi}_t$ represents the observed volatility measures at time $t$. We consider both the annualized standard deviation of daily returns for individual factor returns during the preceding month, denoted by $\sigma_{i,t}$, as well as the implied volatility reported by the CBOE for the S&P 500 on the final trading day of the preceding month, denoted as $VIX_t$. $\theta_{\sigma,i}$ and $\theta_{VIX,i}$ are the vector of sensitivities of each factor to the relevant volatility measure. The first two columns report univariate estimations, where either $\theta_{\sigma,i}$ or $\theta_{VIX,i}$ is set to zero, respectively. The last column shows the multivariate portfolio. The volatility managed multi-factor portfolios have the same unconditional standard deviation as an optimal unmanaged multi-factor portfolio. The optimal unconditional mean-variance factor portfolio's weights are 1.195, 0.448, −0.135, 3.247, 2.070, and 0.942, for MKT, SMB, HML, CMA, RMW, and UMD, respectively. The next part shows results from time-series regressions of each volatility managed multi-factor portfolio on the unconditional factor portfolio: $f_t^\sigma = \alpha + \beta f_t + \epsilon_t$ and Fama and French (2015) 5-factor $\alpha$'s. T-statistics are adjusted for heteroscedasticity and reported in brackets. The last part shows Sharpe ratios for the unmanaged (SR($f$)) and managed portfolios (SR($f^\sigma$)) and the $z$-statistic from the Jobson and Korkie (1981) test of the null that SR($f^\sigma$) − SR($f$) = 0. Alphas and Sharpe ratios are annualized. The sample period is July 1967 to December 2020.

$\alpha$ and $z$-statistic become statistical significance at the 5% level.

Moving to the lower left panel reveals outcomes for high VIX and left-skewed market returns. Compared to the upper left panel, characterized by low implied volatility, we observe only marginal improvements in alphas compared to the untimed portfolio. Though all Sharpe ratios are now larger, none of the Sharpe Ratio improvements due to timing is statistically distinguishable from an untimed portfolio.

In the lower right panel, we report results when implied volatility is high and index options indicate right-skewness. For this subsample, volatility management leads by far to the largest performance improvements. For example, in comparison to an unmanaged portfolio, a volatility managed multi-factor portfolio based on both past factor volatilities and the VIX yields an annualized alpha of 37.1%. Additionally, volatility management enhances the Sharpe ratio by 70%, reaching 1.917 in comparison to the unmanaged portfolio. Moreover, within this risk regime, volatility-managed portfolios deliver the highest FF5 $\alpha$s and the most elevated $z$-statistics for the Jobson and Korkie (1981) test.

### A.2 *Out-of-sample thetas*

### A.3 *Volatility managed multi-factor portfolios with transaction costs*

Here we analyze different levels of trading costs, uniformly applied to transactions involving all stocks underlying the factors. For better clarity, the first panel shows the original results in the

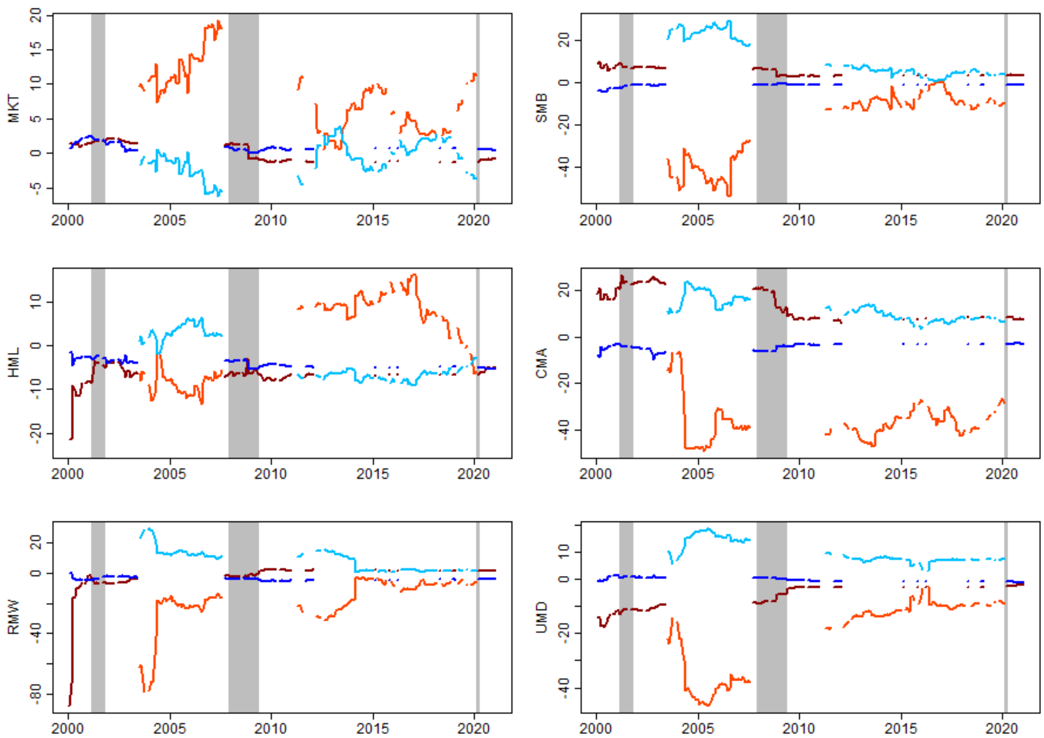


**Figure A.1** Out-of-sample thetas for VIX regime estimates.

Figure A.1 displays the out-of-sample thetas for VIX regime estimates. We color $\theta_\sigma$ and $\theta_{VIX}$ for the high (low) VIX regimes as dark-red (light-red) and dark-blue (light-blue) lines, respectively. The shaded areas represent NBER recession dates. We apply a burn-in period to obtain initial estimates and report results for the time period from 2000 to 2020, where both the VIX and past factor volatilities are used as conditioning variables.

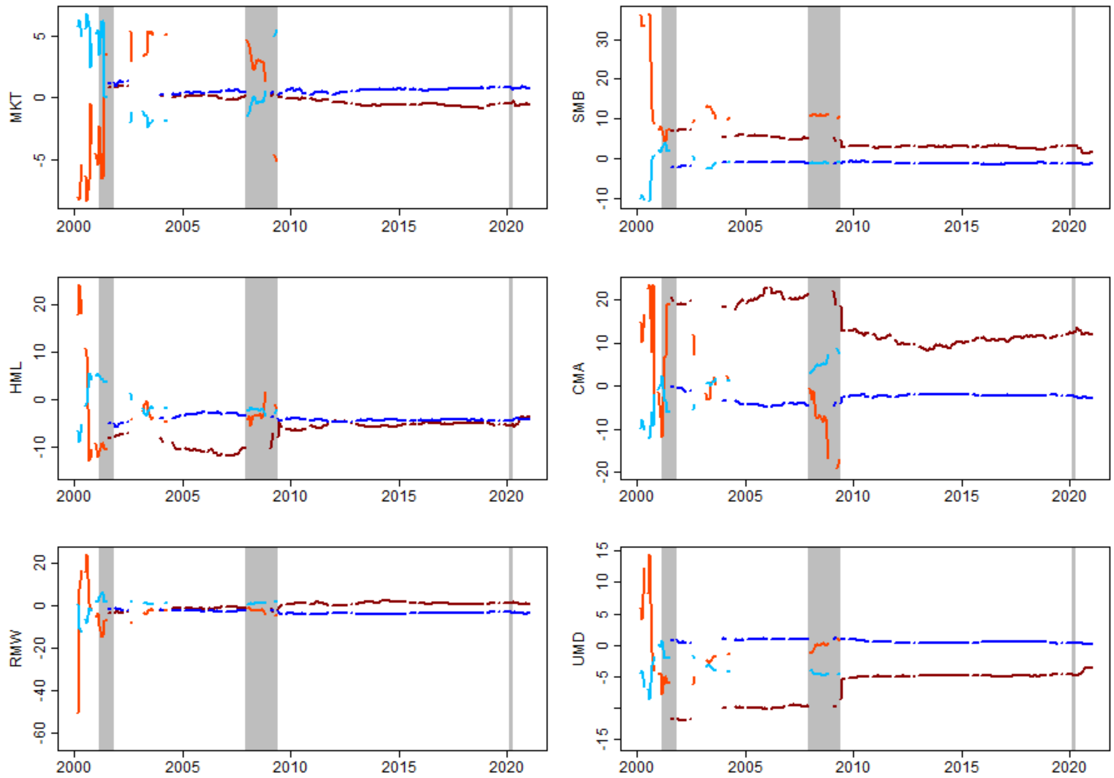


**Figure A.2** Out-of-sample thetas for Skew regime estimates.

Figure A.2 displays the out-of-sample thetas for Skew regime estimates. We color $\theta_\sigma$ and $\theta_{VIX}$ for the left (right) Skewness regimes as dark-red (light-red) and dark-blue (light-blue) lines, respectively. The shaded areas represent NBER recession dates. We apply a burn-in period to obtain initial estimates and report results for the time period from 2000 to 2020, where both the VIX and past factor volatilities are used as conditioning variables.

absence of transaction costs, while the subsequent panels are based on transaction costs of 10, 25, and 50 basis points per round trip. It can be seen that all alphas remain highly significant, and volatility management consistently increases Sharpe ratios across all the considered levels of transaction costs. In the final part of the table, we show that the critical level of transaction costs required to drive the alphas to zero is roughly 151 and 154 basis points, respectively.

Break-even costs for estimates that do not distinguish between different risk regimes range from 139 and 141 basis points when employing the VIX or each factor's past volatility as conditioning variables, respectively. Again, the multivariate approach utilizing both volatility measures delivers the best results: break-even transaction costs are roughly 144 basis points.

## *A.4 Volatility management in a large factor universe*

Up to this point, the analysis has concentrated on volatility management of a portfolio with exposures to FF-5 factors plus momentum. A natural question that may arise is whether the findings are specific to this particular factor set. To address this concern, we broaden the analysis by including a more extensive set of factors. We hereby start with the universe of 206 factors provided by Chen and Zimmermann (2022). Among these factors, 137 are available for our sample period, spanning from July 1967 to December 2020. A principal component analysis (PCA) reveals that the first five PCs capture approximately 70% of the variation of these factor returns. To maintain tractability in the analysis, we focus on volatility-managed portfolios that incorporate exposures to these five principal components, augmented by the market factor.

Table A.10 summarizes the results. The first column presents results, when MKT and each PC's past return volatilities are used as conditioning variables. Similar to our base-case, the MKT receives a positive, passive non-managed weight, which decreases when volatility is high. The resulting alphas, relative to the unmanaged

**Table A.2** Volatility managed multi-factor portfolios for different regimes.

| | VIX regimes | | | Skewness regimes | | |
|---|---|---|---|---|---|---|
| | $\sigma_{i,t}$ | $VIX_t$ | $\sigma_{i,t}\&VIX_t$ | $\sigma_{i,t}$ | $VIX_t$ | $\sigma_{i,t}\&VIX_t$ |
| $\mu_{MKT}$ | 0.981 | 0.802 | 0.959 | 1.341 | 0.987 | 1.032 |
| $\mu_{SMB}$ | 0.049 | 0.662 | 0.286 | −0.148 | 0.605 | 0.344 |
| $\mu_{HML}$ | 1.596 | 1.671 | 2.019 | 1.668 | 1.878 | 1.987 |
| $\mu_{CMA}$ | 2.268 | 2.681 | 2.135 | 2.163 | 2.637 | 1.354 |
| $\mu_{RMW}$ | 2.281 | 3.538 | 3.357 | 2.428 | 3.629 | 3.300 |
| $\mu_{UMD}$ | 1.509 | 1.188 | 1.326 | 1.748 | 1.337 | 1.824 |
| | Low VIX | | | Left skewed | | |
| $\theta_{\sigma,MKT}$ | 13.386 | | 18.435 | −1.960 | | 10.211 |
| $\theta_{\sigma,SMB}$ | −1.759 | | −14.118 | 4.969 | | −4.076 |
| $\theta_{\sigma,HML}$ | −12.085 | | −12.247 | −19.538 | | −2.908 |
| $\theta_{\sigma,CMA}$ | −15.392 | | −42.796 | 11.609 | | 13.091 |
| $\theta_{\sigma,RMW}$ | 11.627 | | −7.196 | −3.330 | | 4.417 |
| $\theta_{\sigma,UMD}$ | 3.006 | | −15.057 | −7.730 | | −7.423 |
| $\theta_{VIX,MKT}$ | | 9.939 | −1.393 | | 0.825 | 1.699 |
| $\theta_{VIX,SMB}$ | | −3.758 | 5.576 | | −2.519 | −2.780 |
| $\theta_{VIX,HML}$ | | −6.344 | −8.211 | | −13.072 | −9.582 |
| $\theta_{VIX,CMA}$ | | −7.305 | 11.583 | | −3.034 | −5.858 |
| $\theta_{VIX,RMW}$ | | −4.415 | 2.395 | | −6.821 | −8.025 |
| $\theta_{VIX,UMD}$ | | 4.829 | −3.427 | | −3.254 | 0.317 |
| | High VIX | | | Right skewed | | |
| $\theta_{\sigma,MKT}$ | −1.078 | | −5.111 | −1.129 | | −10.338 |
| $\theta_{\sigma,SMB}$ | 5.670 | | 4.991 | 13.395 | | 21.554 |
| $\theta_{\sigma,HML}$ | −16.966 | | −3.025 | −6.017 | | −2.386 |
| $\theta_{\sigma,CMA}$ | 7.618 | | 10.424 | −16.505 | | −33.958 |
| $\theta_{\sigma,RMW}$ | −2.208 | | 1.594 | −1.072 | | −9.649 |
| $\theta_{\sigma,UMD}$ | −5.565 | | 13.673 | −4.016 | | 2.646 |
| $\theta_{VIX,MKT}$ | | −0.089 | 0.789 | | 0.236 | −1.181 |
| $\theta_{VIX,SMB}$ | | −0.048 | −1.528 | | 2.965 | 3.126 |
| $\theta_{VIX,HML}$ | | −11.667 | −8.507 | | −9.643 | −7.697 |
| $\theta_{VIX,CMA}$ | | −2.651 | −4.767 | | −3.602 | 25.697 |
| $\theta_{VIX,RMW}$ | | −5.694 | −6.730 | | −4.386 | 2.221 |
| $\theta_{VIX,UMD}$ | | −3.836 | −1.658 | | −3.603 | −7.955 |

**Table A.2** (*Continued*)

| | VIX regimes | | | Skewness regimes | | |
|---|---|---|---|---|---|---|
| | $\sigma_{i,t}$ | $VIX_t$ | $\sigma_{i,t}\&VIX_t$ | $\sigma_{i,t}$ | $VIX_t$ | $\sigma_{i,t}\&VIX_t$ |
| $\alpha$ | 0.177 | 0.228 | 0.260 | 0.149 | 0.199 | 0.273 |
| | (6.258) | (7.397) | (8.177) | (5.455) | (6.354) | (7.726) |
| $\beta$ | 0.756 | 0.700 | 0.668 | 0.789 | 0.731 | 0.655 |
| | (14.652) | (14.774) | (14.372) | (16.165) | (15.671) | (12.005) |
| FF5 $\alpha$ | 0.238 | 0.285 | 0.313 | 0.213 | 0.258 | 0.325 |
| | (8.007) | (9.296) | (9.775) | (7.274) | (8.237) | (9.571) |
| SR($f$) | 1.247 | 1.247 | 1.247 | 1.247 | 1.247 | 1.247 |
| SR($f^\sigma$) | 1.650 | 1.784 | 1.869 | 1.580 | 1.706 | 1.905 |
| z($\Delta$SR) | 3.921 | 4.674 | 5.132 | 3.496 | 4.242 | 5.320 |

This table shows results for volatility managed multi-factor portfolios for different regimes. The left part reports estimates for implied volatility regimes, the right part shows estimates for implied skewness regimes. We split the sample at the median implied volatility and implied skewness, and estimate optimal weights with respect to these thresholds. We estimate the vector of weights, $\boldsymbol{\theta}$, separately for high and low VIX regimes. In this case factor weights are defined by $\boldsymbol{w}_{VIX,t} = \boldsymbol{\mu} + \boldsymbol{I}_v\boldsymbol{\theta}_{Low}^\top\boldsymbol{\Phi}_t + (1 - \boldsymbol{I}_v)\boldsymbol{\theta}_{High}^\top\boldsymbol{\Phi}_t$, where $\boldsymbol{I}_v$ is an indicator function that takes on the value of 1 if the VIX indicates a low-VIX regime at time $t$ and zero otherwise. Similarly, we estimate the vector of weights, $\boldsymbol{\theta}$, separately for left and right-skewness regimes. In this case factor weights are defined by $\boldsymbol{w}_{skew,t} = \boldsymbol{\mu} + \boldsymbol{I}_s\boldsymbol{\theta}_{Left}^\top\boldsymbol{\Phi}_t + (1 - \boldsymbol{I}_s)\boldsymbol{\theta}_{Right}^\top\boldsymbol{\Phi}_t$, where $\boldsymbol{I}_s$ is an indicator function that takes the value of 1 if the skewness measure indicates a left-skewed regime at time $t$ and zero otherwise. $\boldsymbol{\mu}$ represents the vector of passive, non-managed weights for each factor, $\boldsymbol{\theta}$ are coefficients to be estimated representing the sensitivities of each factor weight to the relevant volatility measures, and $\boldsymbol{\Phi}_t$ represents the observed volatility measures at time $t$. We consider both the annualized standard deviation of daily returns for individual factor returns during the preceding month, denoted by $\sigma_{i,t}$, as well as the implied volatility reported by the CBOE for the S&P 500 on the final trading day of the preceding month, denoted as $VIX_t$. $\theta_{\sigma,i}$ and $\theta_{VIX,i}$ are the vector of sensitivities of each factor to the relevant volatility measure. The first two columns report univariate estimations, where either $\theta_{\sigma,i}$ or $\theta_{VIX,i}$ is set to zero, respectively. The last column shows the multivariate portfolio. The first panel shows passive, non-managed weights for each individual factor ($\mu$), followed by optimal sensitivities of each factor to the relevant volatility measure ($\theta$), respectively. The volatility managed multi-factor portfolios have the same unconditional standard deviation as an optimal unmanaged multi-factor portfolio. The optimal unconditional mean-variance factor portfolio's weights are 1.195, 0.448, −0.135, 3.247, 2.070, and 0.942, for MKT, SMB, HML, CMA, RMW, and UMD, respectively. The next part shows results from time-series regressions of each volatility managed multi-factor portfolio on the unconditional factor portfolio: $f_t^\sigma = \alpha + \beta f_t + \epsilon_t$ and Fama and French (2015) 5-factor $\alpha$'s. T-statistics are adjusted for heteroscedasticity and reported in brackets. The last part shows Sharpe ratios for the unmanaged (SR($f$)) and managed portfolios (SR($f^\sigma$)) and the $z$-statistic from the Jobson and Korkie (1981) test of the null that SR($f^\sigma$) − SR($f$) = 0. Alphas and Sharpe ratios are annualized. The sample period is July 1967 to December 2020.

portfolio, as well as the alphas measured against the FF5 model, are positive and highly significant. However, the increase in Sharpe ratio is not significantly different from zero, indicated by a $z$-statistic of 1.568. The next column displays results with the VIX as the relevant volatility measure. Here, the non-managed weights of the MKT and first three PC's are much larger compared to the case with individual volatilities. This leads to a substantially higher alpha and Sharpe ratio. Ultimately, the most favorable results are once again obtained by combining both volatility measures. We find an annualized alpha of 0.128 and a statistically significant increase in the Sharpe ratio by almost 80%. In summary, the robustness of the results persists when applied to principal components derived from a large universe of factors.

**Table A.3** Volatility managed multi-factor portfolios out-of-sample.

| | In-sample | | | Out-of-sample | | |
|---|---|---|---|---|---|---|
| | $\sigma_{i,t}$ | $VIX_t$ | $\sigma_{i,t}\&VIX_t$ | $\sigma_{i,t}$ | $VIX_t$ | $\sigma_{i,t}\&VIX_t$ |
| $\mu_{MKT}$ | 1.332 | 1.009 | 1.237 | 0.624 | 1.274 | 0.672 |
| $\mu_{SMB}$ | −0.096 | 0.616 | 0.322 | −0.045 | 0.972 | 0.213 |
| $\mu_{HML}$ | 1.792 | 1.975 | 2.255 | 1.168 | 2.676 | 1.504 |
| $\mu_{CMA}$ | 1.945 | 2.634 | 2.142 | 1.162 | 3.773 | 1.354 |
| $\mu_{RMW}$ | 2.470 | 3.688 | 3.650 | 1.829 | 5.310 | 2.677 |
| $\mu_{UMD}$ | 1.707 | 1.438 | 1.590 | 1.280 | 1.745 | 1.285 |
| $\theta_{\sigma,MKT}$ | −1.690 | | −2.284 | 0.120 | | −0.640 |
| $\theta_{\sigma,SMB}$ | 6.265 | | 4.276 | 5.542 | | 3.832 |
| $\theta_{\sigma,HML}$ | −18.358 | | −8.593 | −9.307 | | −6.983 |
| $\theta_{\sigma,CMA}$ | 11.320 | | 12.151 | 7.016 | | 12.207 |
| $\theta_{\sigma,RMW}$ | −2.767 | | 1.313 | −5.733 | | −3.355 |
| $\theta_{\sigma,UMD}$ | −6.092 | | −4.725 | −6.411 | | −6.611 |
| $\theta_{VIX,MKT}$ | | 0.284 | 1.594 | | 2.028 | 1.473 |
| $\theta_{VIX,SMB}$ | | −0.248 | −1.270 | | −0.309 | −0.443 |
| $\theta_{VIX,HML}$ | | −12.698 | −9.265 | | −15.619 | −4.365 |
| $\theta_{VIX,CMA}$ | | −2.462 | −4.721 | | −2.219 | −3.729 |
| $\theta_{VIX,RMW}$ | | −5.797 | −6.884 | | −9.356 | −3.976 |
| $\theta_{VIX,UMD}$ | | −3.924 | −0.820 | | −3.599 | 0.327 |
| $\alpha$ | 0.172 | 0.172 | 0.220 | 0.282 | 0.197 | 0.297 |
| | (3.302) | (3.979) | (4.189) | (2.245) | (2.971) | (2.323) |
| $\beta$ | 0.684 | 0.579 | 0.551 | −0.005 | 0.772 | 0.027 |
| | (7.984) | (10.040) | (8.399) | (−0.031) | (10.247) | (0.172) |
| FF5 $\alpha$ | 0.133 | 0.109 | 0.154 | 0.208 | 0.089 | 0.215 |
| | (2.937) | (3.331) | (3.653) | (1.651) | (1.216) | (1.604) |
| SR($f$) | 0.934 | 0.934 | 0.934 | 0.752 | 0.752 | 0.752 |
| SR($f^\sigma$) | 1.263 | 1.418 | 1.475 | 0.700 | 1.072 | 0.760 |
| z($\Delta$SR) | 1.878 | 2.839 | 2.776 | −0.167 | 2.100 | 0.026 |

This table shows results for volatility managed multi-factor portfolios in- and out-of-sample, where the vector of factor weights at time $t$, $\boldsymbol{w}_t$ is given by $\boldsymbol{w}_t = \boldsymbol{\mu}+\boldsymbol{\theta}^\top \boldsymbol{\Phi}_t$. $\boldsymbol{\mu}$ represents the vector of passive, non-managed weights for each factor, $\boldsymbol{\theta}$ are coefficients to be estimated representing the sensitivities of each factor weight to the relevant volatility measures, and $\boldsymbol{\Phi}_t$ represents the observed volatility measures at time $t$. We consider both the annualized standard deviation of daily returns for individual factor returns during the preceding month, denoted by $\sigma_{i,t}$, as well as the implied volatility reported by the CBOE for the S&P 500 on the final trading day of the preceding month, denoted as $VIX_t$. $\theta_{\sigma,i}$ and $\theta_{VIX,i}$ are the vector of sensitivities of each factor to the relevant volatility measure. The first two columns report univariate estimations, where either $\theta_{\sigma,i}$ or $\theta_{VIX,i}$ is set to zero, respectively. The last column shows the multivariate portfolio. The first panel shows passive, non-managed weights for each individual factor ($\mu$), followed by optimal sensitivities of each factor to the relevant volatility measure ($\theta$), respectively. The volatility managed multi-factor portfolios have the same unconditional standard deviation as an optimal unmanaged multi-factor portfolio. The optimal in-sample (average out-of-sample) unconditional mean-variance factor portfolio's weights are 1.195 (1.262), 0.448 (0.631), −0.135 (0.366), 3.247 (3.271), 2.070 (2.085), and 0.942 (1.132), for MKT, SMB, HML, CMA, RMW, and UMD, respectively. The next part shows results from time-series regressions of each volatility managed multi-factor portfolio on the unconditional factor portfolio: $f_t^\sigma = \alpha + \beta f_t + \epsilon_t$ and Fama and French (2015) 5-factor $\alpha$'s. T-statistics are adjusted for heteroscedasticity and reported in brackets. The last part shows Sharpe ratios for the unmanaged (SR($f$)) and managed portfolios (SR($f^\sigma$)) and the $z$-statistic from the Jobson and Korkie (1981) test of the null that SR($f^\sigma$) − SR($f$) = 0. Alphas and Sharpe ratios are annualized. The sample period is January 2000 to December 2020.

**Table A.4** Volatility managed multi-factor portfolios for different regimes out-of-sample.

| | In-sample estimates | | | | | | Out-of-sample estimates | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\sigma_{i,t}$ | $VIX_t$ | $\sigma_{i,t}\&VIX_t$ | $\sigma_{i,t}$ | $VIX_t$ | $\sigma_{i,t}\&VIX_t$ | $\sigma_{i,t}$ | $VIX_t$ | $\sigma_{i,t}\&VIX_t$ | $\sigma_{i,t}$ | $VIX_t$ | $\sigma_{i,t}\&VIX_t$ |
| $\mu_{MKT}$ | 0.982 | 0.803 | 0.960 | 1.343 | 0.988 | 1.034 | 0.496 | 1.012 | 0.535 | 0.662 | 1.049 | 0.429 |
| $\mu_{SMB}$ | 0.049 | 0.663 | 0.286 | −0.148 | 0.606 | 0.344 | −0.035 | 0.853 | 0.122 | −0.129 | 0.797 | 0.032 |
| $\mu_{HML}$ | 1.599 | 1.674 | 2.022 | 1.671 | 1.881 | 1.990 | 1.038 | 2.064 | 1.320 | 1.253 | 2.158 | 1.118 |
| $\mu_{CMA}$ | 2.272 | 2.685 | 2.138 | 2.167 | 2.642 | 1.356 | 1.291 | 3.451 | 1.235 | 1.232 | 3.126 | 0.669 |
| $\mu_{RMW}$ | 2.285 | 3.543 | 3.363 | 2.431 | 3.635 | 3.306 | 1.686 | 4.509 | 2.328 | 1.841 | 4.381 | 1.755 |
| $\mu_{UMD}$ | 1.511 | 1.190 | 1.328 | 1.751 | 1.339 | 1.827 | 1.218 | 1.436 | 1.160 | 1.400 | 1.382 | 1.134 |
| | Low VIX | | | Left skewed | | | Low VIX | | | Left skewed | | |
| $\theta_{\sigma,MKT}$ | 13.407 | | 18.464 | −1.963 | | 10.227 | 7.118 | | 8.436 | −0.445 | | 2.647 |
| $\theta_{\sigma,SMB}$ | −1.762 | | −14.140 | 4.976 | | −4.082 | 5.766 | | −20.530 | 5.915 | | −1.561 |
| $\theta_{\sigma,HML}$ | −12.104 | | −12.266 | −19.568 | | −2.913 | −2.332 | | −0.010 | −13.114 | | −1.462 |
| $\theta_{\sigma,CMA}$ | −15.416 | | −42.862 | 11.628 | | 13.112 | −8.589 | | −30.265 | 9.379 | | 3.576 |
| $\theta_{\sigma,RMW}$ | 11.645 | | −7.207 | −3.336 | | 4.424 | 4.842 | | −27.260 | −6.343 | | 1.446 |
| $\theta_{\sigma,UMD}$ | 3.010 | | −15.081 | −7.742 | | −7.435 | −4.110 | | −20.970 | −8.793 | | −3.727 |
| $\theta_{VIX,MKT}$ | | 9.955 | −1.396 | | 0.826 | 1.702 | | 10.715 | −0.136 | | 1.358 | 0.630 |
| $\theta_{VIX,SMB}$ | | −3.763 | 5.585 | | −2.523 | −2.785 | | 0.756 | 4.895 | | −3.391 | −1.321 |
| $\theta_{VIX,HML}$ | | −6.354 | −8.224 | | −13.092 | −9.597 | | −3.785 | −6.719 | | −14.411 | −4.153 |
| $\theta_{VIX,CMA}$ | | −7.316 | 11.601 | | −3.038 | −5.868 | | −7.538 | 13.192 | | −0.968 | −2.713 |
| $\theta_{VIX,RMW}$ | | −4.422 | 2.399 | | −6.831 | −8.038 | | −1.553 | −1.346 | | −8.843 | −2.803 |
| $\theta_{VIX,UMD}$ | | 4.836 | −3.432 | | −3.259 | 0.317 | | 3.009 | −5.807 | | −2.350 | 0.627 |
| | High VIX | | | Right skewed | | | High VIX | | | Right skewed | | |
| $\theta_{\sigma,MKT}$ | −1.080 | | −5.119 | −1.131 | | −10.354 | 0.372 | | −1.254 | 1.454 | | −1.450 |
| $\theta_{\sigma,SMB}$ | 5.679 | | 4.999 | 13.415 | | 21.588 | 5.327 | | 12.513 | 10.265 | | 11.368 |
| $\theta_{\sigma,HML}$ | −16.992 | | −3.029 | −6.026 | | −2.390 | −8.820 | | −2.454 | −4.485 | | −2.558 |
| $\theta_{\sigma,CMA}$ | 7.630 | | 10.440 | −16.531 | | −34.011 | 5.710 | | 10.678 | −6.295 | | −8.614 |
| $\theta_{\sigma,RMW}$ | −2.212 | | 1.596 | −1.073 | | −9.664 | −4.877 | | 11.166 | −2.282 | | −3.554 |
| $\theta_{\sigma,UMD}$ | −5.574 | | 13.694 | −4.023 | | 2.650 | −6.112 | | 10.076 | −5.038 | | 0.107 |
| $\theta_{VIX,MKT}$ | | −0.089 | 0.790 | | 0.236 | −1.183 | | 0.920 | 0.841 | | 1.760 | −0.098 |
| $\theta_{VIX,SMB}$ | | −0.048 | −1.530 | | 2.969 | 3.130 | | −0.872 | −1.061 | | 3.068 | 4.108 |
| $\theta_{VIX,HML}$ | | −11.685 | −8.521 | | −9.658 | −7.709 | | −13.435 | −4.201 | | −10.033 | −7.356 |
| $\theta_{VIX,CMA}$ | | −2.655 | −4.775 | | −3.608 | 25.737 | | −2.744 | −4.223 | | −4.872 | 14.955 |
| $\theta_{VIX,RMW}$ | | −5.702 | −6.741 | | −4.393 | 2.225 | | −8.584 | −4.149 | | −6.244 | −1.873 |
| $\theta_{VIX,UMD}$ | | −3.842 | −1.661 | | −3.609 | −7.967 | | −3.544 | −0.097 | | −2.850 | −7.440 |
| $\alpha$ | 0.277 | 0.285 | 0.326 | 0.205 | 0.206 | 0.314 | 0.324 | 0.301 | 0.318 | 0.207 | 0.143 | 0.234 |
| | (4.972) | (5.596) | (5.748) | (3.712) | (4.096) | (5.243) | (2.632) | (4.417) | (2.514) | (2.146) | (2.134) | (1.990) |
| $\beta$ | 0.662 | 0.547 | 0.521 | 0.672 | 0.574 | 0.484 | 0.027 | 0.661 | 0.014 | 0.050 | 0.738 | −0.022 |
| | (7.469) | (8.447) | (7.890) | (8.220) | (8.434) | (6.083) | (0.178) | (7.090) | (0.091) | (0.279) | (7.937) | (−0.128) |
| FF5 $\alpha$ | 0.231 | 0.227 | 0.262 | 0.168 | 0.147 | 0.251 | 0.250 | 0.213 | 0.242 | 0.129 | 0.062 | 0.181 |
| | (4.424) | (5.263) | (5.298) | (3.437) | (3.716) | (4.978) | (1.994) | (2.757) | (1.785) | (1.471) | (0.818) | (1.447) |
| SR($f$) | 0.934 | 0.934 | 0.934 | 0.934 | 0.934 | 0.934 | 0.752 | 0.752 | 0.752 | 0.752 | 0.752 | 0.752 |
| SR($f^\sigma$) | 1.551 | 1.730 | 1.763 | 1.328 | 1.480 | 1.739 | 0.828 | 1.247 | 0.803 | 0.554 | 0.912 | 0.568 |
| z($\Delta$SR) | 3.210 | 4.001 | 3.848 | 2.132 | 2.982 | 3.654 | 0.245 | 2.648 | 0.163 | −0.652 | 0.986 | −0.586 |

This table shows results for volatility managed multi-factor portfolios for different regimes in- and out-of-sample. The left part reports the in-sample estimates, the right part shows out-of-sample estimates. We split the sample at the (expanding) median implied volatility and implied skewness, and estimate optimal weights with respect to these thresholds. We estimate the vector of weights, $\boldsymbol{\theta}$, separately for high and low VIX regimes. In this case factor weights are defined by $\boldsymbol{w}_{VIX,t} = \boldsymbol{\mu} + \boldsymbol{I}_v\boldsymbol{\theta}_{Low}^\top\boldsymbol{\Phi}_t + (1-\boldsymbol{I}_v)\boldsymbol{\theta}_{High}^\top\boldsymbol{\Phi}_t$, where $\boldsymbol{I}_v$ is an indicator function that takes on the value of 1 if the VIX indicates a low-VIX regime at time $t$ and zero otherwise. Similarly, we estimate the vector of weights, $\boldsymbol{\theta}$, separately for left and right-skewness regimes. In this case factor weights are defined by $\boldsymbol{w}_{skew,t} = \boldsymbol{\mu} + \boldsymbol{I}_s\boldsymbol{\theta}_{Left}^\top\boldsymbol{\Phi}_t + (1-\boldsymbol{I}_s)\boldsymbol{\theta}_{Right}^\top\boldsymbol{\Phi}_t$, where $\boldsymbol{I}_s$ is an indicator function that takes the value of 1 if the skewness measure indicates a left-skewed regime at time $t$ and zero otherwise. $\boldsymbol{\mu}$ represents the vector of passive, non-managed weights for each factor, $\boldsymbol{\theta}$ are coefficients to be estimated representing the sensitivities of each factor weight to the relevant volatility measures, and $\boldsymbol{\Phi}_t$ represents the observed volatility measures at time $t$. We consider both the annualized standard deviation of daily returns for individual factor returns during the preceding month, denoted by $\sigma_{i,t}$, as well as the implied volatility reported by the CBOE for the S&P 500 on the final trading day of the preceding month, denoted as $VIX_t$. $\theta_{\sigma,i}$ and $\theta_{VIX,i}$ are the vector of sensitivities of each factor to the relevant volatility measure. The first two columns report univariate estimations, where either $\theta_{\sigma,i}$ or $\theta_{VIX,i}$ is set to zero, respectively. The last column shows the multivariate portfolio. The first panel shows passive, non-managed weights for each individual factor ($\mu$), followed by optimal sensitivities of each factor to the relevant volatility measure ($\theta$), respectively. The volatility managed multi-factor portfolios have the same unconditional standard deviation as an optimal unmanaged multi-factor portfolio. The optimal in-sample (average out-of-sample) unconditional mean-variance factor portfolio's weights are 1.195 (1.262), 0.448 (0.631), −0.135 (0.366), 3.247 (3.271), 2.070 (2.085), and 0.942 (1.132), for MKT, SMB, HML, CMA, RMW, and UMD, respectively. The next part shows results from time-series regressions of each volatility managed multi-factor portfolio on the unconditional factor portfolio: $f_t^\sigma = \alpha + \beta f_t + \epsilon_t$ and Fama and French (2015) 5-factor $\alpha$'s. T-statistics are adjusted for heteroscedasticity and reported in brackets. The last part shows Sharpe ratios for the unmanaged (SR($f$)) and managed portfolios (SR($f^\sigma$)) and the $z$-statistic from the Jobson and Korkie (1981) test of the null that SR($f^\sigma$) − SR($f$) = 0. Alphas and Sharpe ratios are annualized. The sample period is January 2000 to December 2020.

**Table A.5** Volatility managed multi-factor portfolios for different regimes with different levels of transaction costs.

| | VIX regimes | Skewness regimes |
|---|---|---|
| **Panel A: No Transaction costs** | | |
| $\alpha$ | 0.260 | 0.273 |
| | (8.177) | (7.726) |
| FF5 $\alpha$ | 0.313 | 0.325 |
| | (9.775) | (9.571) |
| SR($f$) | 1.247 | 1.247 |
| SR($f^{\sigma}$) | 1.869 | 1.905 |
| z($\Delta$SR) | 5.132 | 5.320 |
| **Panel B: Transaction costs: 10 BP** | | |
| $\alpha$ | 0.246 | 0.261 |
| | (7.935) | (7.657) |
| FF5 $\alpha$ | 0.279 | 0.294 |
| | (8.725) | (8.680) |
| SR($f$) | 1.129 | 1.129 |
| SR($f^{\sigma}$) | 1.737 | 1.786 |
| z($\Delta$SR) | 5.070 | 5.352 |
| **Panel C: Transaction costs: 25 BP** | | |
| $\alpha$ | 0.224 | 0.245 |
| | (7.524) | (7.522) |
| FF5 $\alpha$ | 0.228 | 0.248 |
| | (7.131) | (7.327) |
| SR($f$) | 0.952 | 0.952 |
| SR($f^{\sigma}$) | 1.536 | 1.604 |
| z($\Delta$SR) | 4.957 | 5.388 |
| **Panel D: Transaction costs: 50 BP** | | |
| $\alpha$ | 0.189 | 0.215 |
| | (6.702) | (7.183) |
| FF5 $\alpha$ | 0.144 | 0.171 |
| | (4.452) | (5.041) |
| SR($f$) | 0.655 | 0.655 |
| SR($f^{\sigma}$) | 1.195 | 1.294 |
| z($\Delta$SR) | 4.708 | 5.402 |
| **Panel E: Break-even BP** | | |
| | 151.293 | 154.224 |

This table shows performance statistics for the volatility managed multi-factor portfolios for regime estimates with different levels of transaction costs. In panel B, C and D we assume round trip transaction costs for each individual firm underlying the factors of 10, 25, and 50 basis points, respectively. Panel E shows the break-even transaction costs that would drive our alphas to zero.

**Table A.6** Volatility managed multi-factor portfolios with transaction costs.

| | In-sample | | | Out-of-sample | | |
|---|---|---|---|---|---|---|
| | $\sigma_{i,t}$ | $VIX_t$ | $\sigma_{i,t}\&VIX_t$ | $\sigma_{i,t}$ | $VIX_t$ | $\sigma_{i,t}\&VIX_t$ |
| $\mu_{MKT}$ | 0.824 | 0.503 | 0.657 | 0.771 | 0.476 | 0.663 |
| $\mu_{SMB}$ | −0.267 | 0.000 | −0.178 | −0.074 | 0.000 | −0.074 |
| $\mu_{HML}$ | 0.635 | 1.034 | 0.906 | 0.622 | 1.038 | 0.898 |
| $\mu_{CMA}$ | −0.052 | 0.000 | −0.025 | −0.003 | 0.000 | −0.021 |
| $\mu_{RMW}$ | −0.020 | 0.256 | 0.151 | 0.107 | 0.166 | 0.204 |
| $\mu_{UMD}$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $\theta_{\sigma,MKT}$ | −1.136 | | −1.711 | −1.412 | | −1.664 |
| $\theta_{\sigma,SMB}$ | 4.262 | | 3.161 | 1.415 | | 1.426 |
| $\theta_{\sigma,HML}$ | −4.237 | | 0.000 | −0.582 | | 0.887 |
| $\theta_{\sigma,CMA}$ | 2.761 | | 0.928 | 0.296 | | 0.826 |
| $\theta_{\sigma,RMW}$ | 5.609 | | 1.557 | 0.008 | | 0.366 |
| $\theta_{\sigma,UMD}$ | 0.000 | | 0.000 | 0.000 | | 0.000 |
| $\theta_{VIX,MKT}$ | | 0.485 | 1.485 | | 0.927 | 1.593 |
| $\theta_{VIX,SMB}$ | | 0.301 | −0.004 | | 0.594 | 0.325 |
| $\theta_{VIX,HML}$ | | −5.286 | −4.724 | | −4.229 | −3.983 |
| $\theta_{VIX,CMA}$ | | 0.527 | 0.430 | | 0.406 | 0.323 |
| $\theta_{VIX,RMW}$ | | 2.385 | 2.368 | | 2.193 | 1.502 |
| $\theta_{VIX,UMD}$ | | 0.000 | 0.000 | | 0.005 | 0.001 |
| $\alpha$ | 0.030 | 0.045 | 0.056 | 0.008 | 0.031 | 0.028 |
| | (2.270) | (2.661) | (3.116) | (0.398) | (1.575) | (1.306) |
| $\beta$ | 0.783 | 0.733 | 0.724 | 0.579 | 0.673 | 0.639 |
| | (12.258) | (7.291) | (8.565) | (5.127) | (6.615) | (6.874) |
| FF5 $\alpha$ | 0.012 | 0.010 | 0.019 | 0.012 | −0.012 | −0.006 |
| | (1.081) | (0.916) | (1.598) | (0.492) | (−0.865) | (−0.370) |
| SR($f$) | 0.611 | 0.611 | 0.611 | 0.368 | 0.368 | 0.368 |
| SR($f^\sigma$) | 0.829 | 0.901 | 1.014 | 0.288 | 0.519 | 0.479 |
| z(ΔSR) | 1.710 | 1.809 | 2.497 | −0.400 | 0.844 | 0.589 |

This table shows results for volatility managed multi-factor portfolios in- and out-of-sample with transaction costs. The left part reports the in-sample estimates, the right part shows out-of-sample estimates. We split the sample at the (expanding) median implied volatility and implied skewness, and estimate optimal weights with respect to these levels. We estimate the vector of weights, $\boldsymbol{\theta}$, separately for high and low VIX regimes. In this case factor weights are defined by $\boldsymbol{w}_{VIX,t} = \boldsymbol{\mu} + \boldsymbol{I}_v\boldsymbol{\theta}_{Low}^\top\boldsymbol{\Phi}_t + (1-\boldsymbol{I}_v)\boldsymbol{\theta}_{High}^\top\boldsymbol{\Phi}_t$, where $\boldsymbol{I}_v$ is an indicator function that takes on the value of 1 if the VIX indicates a low-VIX regime at time $t$ and zero otherwise. Similarly, we estimate the vector of weights, $\boldsymbol{\theta}$, separately for left and right-skewness regimes. In this case factor weights are defined by $\boldsymbol{w}_{skew,t} = \boldsymbol{\mu} + \boldsymbol{I}_s\boldsymbol{\theta}_{Left}^\top\boldsymbol{\Phi}_t + (1-\boldsymbol{I}_s)\boldsymbol{\theta}_{Right}^\top\boldsymbol{\Phi}_t$, where $\boldsymbol{I}_s$ is an indicator function that takes the value of 1 if the skewness measure indicates a left-skewed regime at time $t$ and zero otherwise. $\boldsymbol{\mu}$ represents the vector of passive, non-managed weights for each factor, $\boldsymbol{\theta}$ are coefficients to be estimated representing the sensitivities of each factor weight to the relevant volatility measures, and $\boldsymbol{\Phi}_t$ represents the observed volatility measures at time $t$. We consider both the annualized standard deviation of daily returns for individual factor returns during the preceding month, denoted by $\sigma_{i,t}$, as well as the implied volatility reported by the CBOE for the S&P 500 on the final trading day of the preceding month, denoted as $VIX_t$. $\theta_{\sigma,i}$ and $\theta_{VIX,i}$ are the vector of sensitivities of each factor to the relevant volatility measure. The first two columns report univariate estimations, where either $\theta_{\sigma,i}$ or $\theta_{VIX,i}$ is set to zero, respectively. The last column shows the multivariate portfolio. The first panel shows passive, non-managed weights for each individual factor ($\mu$), followed by optimal sensitivities of each factor to the relevant volatility measure ($\theta$), respectively. The volatility managed multi-factor portfolios have the same unconditional standard deviation as an optimal unmanaged multi-factor portfolio. The optimal in-sample (average out-of-sample) weights for the unconditional mean-variance factor portfolio with transaction costs are 0.652 (0.633), 0.000 (0.000), 0.184 (0.638), 0.320 (0.047), 0.414 (0.154), and 0.000 (0.000), for MKT, SMB, HML, CMA, RMW, and UMD, respectively. The next part shows results from time-series regressions of each volatility managed multi-factor portfolio on the unconditional factor portfolio: $f_t^\sigma = \alpha + \beta f_t + \epsilon_t$ and Fama and French (2015) 5-factor $\alpha$'s. T-statistics are adjusted for heteroscedasticity and reported in brackets. The last part shows Sharpe ratios for the unmanaged (SR($f$)) and managed portfolios (SR($f^\sigma$)) and the $z$-statistic from the Jobson and Korkie (1981) test of the null that $\text{SR}(f^\sigma) - \text{SR}(f) = 0$. Alphas and Sharpe ratios are annualized. The sample period is January 2000 to December 2020.

**Table A.7** Volatility managed multi-factor portfolios for different regimes with transaction costs.

| | No transaction costs | | | | | | Transaction costs | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\sigma_{i,t}$ | $VIX_t$ | $\sigma_{i,t}$&$VIX_t$ | $\sigma_{i,t}$ | $VIX_t$ | $\sigma_{i,t}$&$VIX_t$ | $\sigma_{i,t}$ | $VIX_t$ | $\sigma_{i,t}$&$VIX_t$ | $\sigma_{i,t}$ | $VIX_t$ | $\sigma_{i,t}$&$VIX_t$ |
| $\mu_{MKT}$ | 0.981 | 0.802 | 0.959 | 1.341 | 0.987 | 1.032 | 0.445 | 0.358 | 0.437 | 0.764 | 0.462 | 0.542 |
| $\mu_{SMB}$ | 0.049 | 0.662 | 0.286 | −0.148 | 0.605 | 0.344 | −0.144 | 0.000 | −0.205 | −0.110 | 0.000 | −0.008 |
| $\mu_{HML}$ | 1.596 | 1.671 | 2.019 | 1.668 | 1.878 | 1.987 | 0.435 | 0.876 | 0.711 | 0.796 | 0.948 | 0.889 |
| $\mu_{CMA}$ | 2.268 | 2.681 | 2.135 | 2.163 | 2.637 | 1.354 | −0.065 | 0.000 | −0.009 | −0.012 | 0.000 | −0.027 |
| $\mu_{RMW}$ | 2.281 | 3.538 | 3.357 | 2.428 | 3.629 | 3.300 | −0.084 | 0.161 | −0.028 | −0.035 | 0.238 | 0.061 |
| $\mu_{UMD}$ | 1.509 | 1.188 | 1.326 | 1.748 | 1.337 | 1.824 | 0.000 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 |
| | Low VIX | | | Left skewed | | | Low VIX | | | Left skewed | | |
| $\theta_{\sigma,MKT}$ | 13.386 | | 18.435 | −1.960 | | 10.211 | 9.651 | | 2.977 | −1.253 | | 4.829 |
| $\theta_{\sigma,SMB}$ | −1.759 | | −14.118 | 4.969 | | −4.076 | 1.857 | | −1.245 | 1.505 | | 0.093 |
| $\theta_{\sigma,HML}$ | −12.085 | | −12.247 | −19.538 | | −2.908 | −0.019 | | 0.890 | −7.394 | | −3.122 |
| $\theta_{\sigma,CMA}$ | −15.392 | | −42.796 | 11.609 | | 13.091 | 2.635 | | 0.599 | 1.888 | | 0.528 |
| $\theta_{\sigma,RMW}$ | 11.627 | | −7.196 | −3.330 | | 4.417 | 24.428 | | 5.106 | 6.265 | | 4.128 |
| $\theta_{\sigma,UMD}$ | 3.006 | | −15.057 | −7.730 | | −7.423 | 4.116 | | 1.759 | 0.000 | | 0.000 |
| $\theta_{VIX,MKT}$ | | 9.939 | −1.393 | | 0.825 | 1.699 | | 6.054 | −1.009 | | 0.528 | 1.324 |
| $\theta_{VIX,SMB}$ | | −3.758 | 5.576 | | −2.519 | −2.780 | | 0.001 | 3.820 | | 0.000 | −0.024 |
| $\theta_{VIX,HML}$ | | −6.344 | −8.211 | | −13.072 | −9.582 | | −3.531 | 0.127 | | −5.949 | −5.166 |
| $\theta_{VIX,CMA}$ | | −7.305 | 11.583 | | −3.034 | −5.858 | | 0.002 | 0.443 | | 0.367 | 0.294 |
| $\theta_{VIX,RMW}$ | | −4.415 | 2.395 | | −6.821 | −8.025 | | 5.765 | 1.781 | | 2.582 | 1.830 |
| $\theta_{VIX,UMD}$ | | 4.829 | −3.427 | | −3.254 | 0.317 | | 2.370 | 0.000 | | 0.000 | 0.000 |
| | High VIX | | | Right skewed | | | High VIX | | | Right skewed | | |
| $\theta_{\sigma,MKT}$ | −1.078 | | −5.111 | −1.129 | | −10.338 | −0.472 | | 3.503 | −0.578 | | −5.103 |
| $\theta_{\sigma,SMB}$ | 5.670 | | 4.991 | 13.395 | | 21.554 | 2.655 | | 1.915 | 8.003 | | 5.959 |
| $\theta_{\sigma,HML}$ | −16.966 | | −3.025 | −6.017 | | −2.386 | −3.061 | | −2.517 | −3.085 | | 1.130 |
| $\theta_{\sigma,CMA}$ | 7.618 | | 10.424 | −16.505 | | −33.958 | 3.036 | | −0.044 | 0.480 | | −0.232 |
| $\theta_{\sigma,RMW}$ | −2.208 | | 1.594 | −1.072 | | −9.649 | 3.794 | | 5.407 | 7.830 | | −1.360 |
| $\theta_{\sigma,UMD}$ | −5.565 | | 13.673 | −4.016 | | 2.646 | 0.000 | | 1.610 | 0.000 | | 0.000 |
| $\theta_{VIX,MKT}$ | | −0.089 | 0.789 | | 0.236 | −1.181 | | 0.281 | 0.985 | | 0.463 | −1.209 |
| $\theta_{VIX,SMB}$ | | −0.048 | −1.528 | | 2.965 | 3.126 | | 0.412 | −0.234 | | 2.516 | 0.131 |
| $\theta_{VIX,HML}$ | | −11.667 | −8.507 | | −9.643 | −7.697 | | −4.360 | −3.833 | | −3.539 | −1.322 |
| $\theta_{VIX,CMA}$ | | −2.651 | −4.767 | | −3.602 | 25.697 | | 0.480 | 0.489 | | 0.329 | 1.032 |
| $\theta_{VIX,RMW}$ | | −5.694 | −6.730 | | −4.386 | 2.221 | | 1.923 | 2.216 | | 2.966 | 3.967 |
| $\theta_{VIX,UMD}$ | | −3.836 | −1.658 | | −3.603 | −7.955 | | −0.004 | 0.000 | | 0.000 | 0.000 |
| $\alpha$ | 0.177 | 0.228 | 0.260 | 0.149 | 0.199 | 0.273 | 0.036 | 0.049 | 0.054 | 0.021 | 0.045 | 0.055 |
| | (6.258) | (7.397) | (8.177) | (5.455) | (6.354) | (7.726) | (3.805) | (4.694) | (4.960) | (2.453) | (4.184) | (4.648) |
| $\beta$ | 0.756 | 0.700 | 0.668 | 0.789 | 0.731 | 0.655 | 0.709 | 0.639 | 0.614 | 0.808 | 0.654 | 0.599 |
| | (14.652) | (14.774) | (14.372) | (16.165) | (15.671) | (12.005) | (20.046) | (12.927) | (12.307) | (18.370) | (10.447) | (9.472) |
| FF5 $\alpha$ | 0.238 | 0.285 | 0.313 | 0.213 | 0.258 | 0.325 | 0.017 | 0.034 | 0.037 | −0.002 | 0.026 | 0.036 |
| | (8.007) | (9.296) | (9.775) | (7.274) | (8.237) | (9.571) | (1.748) | (3.131) | (3.296) | (−0.177) | (2.424) | (3.056) |
| SR($f$) | 1.247 | 1.247 | 1.247 | 1.247 | 1.247 | 1.247 | 0.478 | 0.478 | 0.478 | 0.478 | 0.478 | 0.478 |
| SR($f^{\sigma}$) | 1.650 | 1.784 | 1.869 | 1.580 | 1.706 | 1.905 | 0.707 | 0.812 | 0.849 | 0.606 | 0.777 | 0.855 |
| z(ΔSR) | 3.921 | 4.674 | 5.132 | 3.496 | 4.242 | 5.320 | 2.159 | 2.827 | 3.036 | 1.494 | 2.589 | 3.025 |

This table shows results for volatility managed multi-factor portfolios for different regimes before and after transaction costs. The left part reports estimates in the absence of transaction costs, the right part shows estimates incorporating transaction costs. We split the sample at the median implied volatility and implied skewness, and estimate optimal weights with respect to these levels. We estimate the vector of weights, $\boldsymbol{\theta}$, separately for high and low VIX regimes. In this case factor weights are defined by $\boldsymbol{w_{VIX,t}} = \boldsymbol{\mu} + \boldsymbol{I_v}\boldsymbol{\theta}_{Low}^{\top}\boldsymbol{\Phi}_t + (1 - \boldsymbol{I_v})\boldsymbol{\theta}_{High}^{\top}\boldsymbol{\Phi}_t$, where $\boldsymbol{I_v}$ is an indicator function that takes on the value of 1 if the VIX indicates a low-VIX regime at time $t$ and zero otherwise. Similarly, we estimate the vector of weights, $\boldsymbol{\theta}$, separately for left and right-skewness regimes. In this case factor weights are defined by $\boldsymbol{w}_{skew,t} = \boldsymbol{\mu} + \boldsymbol{I_S}\boldsymbol{\theta}_{Left}^{\top}\boldsymbol{\Phi}_t + (1 - \boldsymbol{I_S})\boldsymbol{\theta}_{Right}^{\top}\boldsymbol{\Phi}_t$, where $\boldsymbol{I_S}$ is an indicator function that takes the value of 1 if the skewness measure indicates a left-skewed regime at time $t$ and zero otherwise. $\boldsymbol{\mu}$ represents the vector of passive, non-managed weights for each factor, $\boldsymbol{\theta}$ are coefficients to be estimated representing the sensitivities of each factor weight to the relevant volatility measures, and $\boldsymbol{\Phi}_t$ represents the observed volatility measures at time $t$. We consider both the annualized standard deviation of daily returns for individual factor returns during the preceding month, denoted by $\sigma_{i,t}$, as well as the implied volatility reported by the CBOE for the S&P 500 on the final trading day of the preceding month, denoted as $VIX_t$. $\theta_{\sigma,i}$ and $\theta_{VIX,i}$ are the vector of sensitivities of each factor to the relevant volatility measure. The first two columns report univariate estimations, where either $\theta_{\sigma,i}$ or $\theta_{VIX,i}$ is set to zero, respectively. The last column shows the multivariate portfolio. The first panel shows passive, non-managed weights for each individual factor ($\mu$), followed by optimal sensitivities of each factor to the relevant volatility measure ($\theta$), respectively. The volatility managed multi-factor portfolios have the same unconditional standard deviation as an optimal unmanaged multi-factor portfolio. The optimal in-sample (average out-of-sample) weights for the unconditional mean-variance factor portfolio with transaction costs are 0.652 (0.633), 0.000 (0.000), 0.184 (0.638), 0.320 (0.047), 0.414 (0.154), and 0.000 (0.000), for MKT, SMB, HML, CMA, RMW, and UMD, respectively. The next part shows results from time-series regressions of each volatility managed multi-factor portfolio on the unconditional factor portfolio: $f_t^{\sigma} = \alpha + \beta f_t + \epsilon_t$ and Fama and French (2015) 5-factor $\alpha$'s. T-statistics are adjusted for heteroscedasticity and reported in brackets. The last part shows Sharpe ratios for the unmanaged (SR($f$)) and managed portfolios (SR($f^{\sigma}$)) and the $z$-statistic from the Jobson and Korkie (1981) test of the null that SR($f^{\sigma}$) − SR($f$) = 0. Alphas and Sharpe ratios are annualized. The sample period is January 2000 to December 2020.

**Table A.8** Volatility managed multi-factor portfolios with different levels of transaction costs.

| | $\sigma_{i,t}$ | $VIX_t$ | $\sigma_{i,t}\&VIX_t$ |
|---|---|---|---|
| **Panel A: No Transaction costs** | | | |
| $\alpha$ | 0.129 | 0.180 | 0.215 |
| | (4.855) | (6.199) | (6.954) |
| FF5 $\alpha$ | 0.195 | 0.240 | 0.272 |
| | (6.815) | (8.048) | (8.635) |
| SR($f$) | 1.247 | 1.247 | 1.247 |
| SR($f^\sigma$) | 1.531 | 1.656 | 1.748 |
| z($\Delta$SR) | 3.186 | 3.954 | 4.477 |
| **Panel B: Transaction costs: 10 BP** | | | |
| $\alpha$ | 0.122 | 0.172 | 0.206 |
| | (4.743) | (6.103) | (6.843) |
| FF5 $\alpha$ | 0.163 | 0.210 | 0.241 |
| | (5.705) | (7.046) | (7.667) |
| SR($f$) | 1.129 | 1.129 | 1.129 |
| SR($f^\sigma$) | 1.407 | 1.538 | 1.628 |
| z($\Delta$SR) | 3.141 | 3.984 | 4.496 |
| **Panel C: Transaction costs: 25 BP** | | | |
| $\alpha$ | 0.110 | 0.160 | 0.191 |
| | (4.534) | (5.926) | (6.640) |
| FF5 $\alpha$ | 0.114 | 0.164 | 0.195 |
| | (4.021) | (5.521) | (6.198) |
| SR($f$) | 0.952 | 0.952 | 0.952 |
| SR($f^\sigma$) | 1.218 | 1.358 | 1.445 |
| z($\Delta$SR) | 3.054 | 4.016 | 4.510 |
| **Panel D: Transaction costs: 50 BP** | | | |
| $\alpha$ | 0.090 | 0.140 | 0.166 |
| | (4.044) | (5.528) | (6.182) |
| FF5 $\alpha$ | 0.034 | 0.088 | 0.117 |
| | (1.193) | (2.953) | (3.723) |
| SR($f$) | 0.655 | 0.655 | 0.655 |
| SR($f^\sigma$) | 0.896 | 1.053 | 1.135 |
| z($\Delta$SR) | 2.850 | 4.031 | 4.490 |
| Panel E: Break-even BP | | | |
| | 124.040 | 134.144 | 141.592 |

This table shows performance statistics for the volatility managed multi-factor portfolios for different levels of transaction costs. In panel B, C and D we assume round trip transaction costs for each individual firm underlying the factors of 10, 25, and 50 basis points, respectively. Panel E shows the break-even transaction costs that would drive our alphas to zero.

**Table A.9** Volatility managed multi-factor portfolios for different regimes with different levels of transaction costs

| | VIX regime estimates | | | Skew regime estimates | | |
|---|---|---|---|---|---|---|
| | $\sigma_{i,t}$ | $VIX_t$ | $\sigma_{i,t}\&VIX_t$ | $\sigma_{i,t}$ | $VIX_t$ | $\sigma_{i,t}\&VIX_t$ |
| **Panel A: No Transaction costs** | | | | | | |
| $\alpha$ | 0.177 | 0.228 | 0.260 | 0.149 | 0.199 | 0.273 |
| | (6.258) | (7.397) | (8.177) | (5.455) | (6.354) | (7.726) |
| FF5 $\alpha$ | 0.238 | 0.285 | 0.313 | 0.213 | 0.258 | 0.325 |
| | (8.007) | (9.296) | (9.775) | (7.274) | (8.237) | (9.571) |
| SR($f$) | 1.247 | 1.247 | 1.247 | 1.247 | 1.247 | 1.247 |
| SR($f^\sigma$) | 1.650 | 1.784 | 1.869 | 1.580 | 1.706 | 1.905 |
| z($\Delta$SR) | 3.921 | 4.674 | 5.132 | 3.496 | 4.242 | 5.320 |
| **Panel B: Transaction costs: 10 BP** | | | | | | |
| $\alpha$ | 0.167 | 0.217 | 0.246 | 0.142 | 0.192 | 0.261 |
| | (6.085) | (7.207) | (7.935) | (5.343) | (6.303) | (7.657) |
| FF5 $\alpha$ | 0.205 | 0.252 | 0.279 | 0.181 | 0.229 | 0.294 |
| | (6.909) | (8.245) | (8.725) | (6.214) | (7.313) | (8.680) |
| SR($f$) | 1.129 | 1.129 | 1.129 | 1.129 | 1.129 | 1.129 |
| SR($f^\sigma$) | 1.522 | 1.657 | 1.737 | 1.459 | 1.592 | 1.786 |
| z($\Delta$SR) | 3.864 | 4.642 | 5.070 | 3.489 | 4.306 | 5.352 |
| **Panel C: Transaction costs: 25 BP** | | | | | | |
| $\alpha$ | 0.151 | 0.199 | 0.224 | 0.130 | 0.181 | 0.245 |
| | (5.780) | (6.886) | (7.524) | (5.139) | (6.204) | (7.522) |
| FF5 $\alpha$ | 0.156 | 0.203 | 0.228 | 0.134 | 0.185 | 0.248 |
| | (5.246) | (6.649) | (7.131) | (4.604) | (5.911) | (7.327) |
| SR($f$) | 0.952 | 0.952 | 0.952 | 0.952 | 0.952 | 0.952 |
| SR($f^\sigma$) | 1.327 | 1.463 | 1.536 | 1.273 | 1.417 | 1.604 |
| z($\Delta$SR) | 3.761 | 4.581 | 4.957 | 3.460 | 4.394 | 5.388 |
| **Panel D: Transaction costs: 50 BP** | | | | | | |
| $\alpha$ | 0.124 | 0.169 | 0.189 | 0.109 | 0.161 | 0.215 |
| | (5.127) | (6.234) | (6.702) | (4.673) | (5.961) | (7.183) |
| FF5 $\alpha$ | 0.073 | 0.122 | 0.144 | 0.055 | 0.111 | 0.171 |
| | (2.456) | (3.966) | (4.452) | (1.894) | (3.553) | (5.041) |
| SR($f$) | 0.655 | 0.655 | 0.655 | 0.655 | 0.655 | 0.655 |
| SR($f^\sigma$) | 0.998 | 1.137 | 1.195 | 0.958 | 1.123 | 1.294 |
| z($\Delta$SR) | 3.539 | 4.436 | 4.708 | 3.353 | 4.513 | 5.402 |
| **Panel E: Break-even BP** | | | | | | |
| | 133.587 | 144.375 | 151.293 | 127.929 | 138.120 | 154.224 |

This table shows performance statistics for the volatility managed multi-factor portfolios for regime estimates with different levels of transaction costs. In panel B, C and D we assume round trip transaction costs for each individual firm underlying the factors of 10, 25, and 50 basis points, respectively. Panel E shows the break-even transaction costs that would drive our alphas to zero.

**Table A.10** Volatility managed multi-factor portfolios with principal components.

| | $\sigma_{i,t}$ | $VIX_t$ | $\sigma_{i,t}$&$VIX_t$ |
|---|---|---|---|
| $\mu_{MKT}$ | 1.812 | 1.098 | 1.434 |
| $\mu_{PC1}$ | −0.225 | −0.334 | −0.366 |
| $\mu_{PC2}$ | −0.088 | −0.272 | −0.235 |
| $\mu_{PC3}$ | −0.274 | −0.046 | −0.078 |
| $\mu_{PC4}$ | −0.357 | −0.040 | −0.162 |
| $\mu_{PC5}$ | −0.118 | −0.549 | −0.425 |
| $\theta_{\sigma,MKT}$ | −2.413 | | −3.185 |
| $\theta_{\sigma,PC1}$ | 1.852 | | 0.596 |
| $\theta_{\sigma,PC2}$ | −0.976 | | −1.854 |
| $\theta_{\sigma,PC3}$ | 3.258 | | 3.769 |
| $\theta_{\sigma,PC4}$ | 7.871 | | 7.657 |
| $\theta_{\sigma,PC5}$ | 8.494 | | −12.980 |
| $\theta_{VIX,MKT}$ | | 0.201 | 0.953 |
| $\theta_{VIX,PC1}$ | | 0.822 | 0.945 |
| $\theta_{VIX,PC2}$ | | 0.834 | 1.213 |
| $\theta_{VIX,PC3}$ | | −0.220 | −0.729 |
| $\theta_{VIX,PC4}$ | | −1.008 | −0.832 |
| $\theta_{VIX,PC5}$ | | 3.171 | 3.398 |
| $\alpha$ | 0.030 | 0.099 | 0.128 |
| | (2.269) | (6.214) | (7.356) |
| $\beta$ | 0.866 | 0.637 | 0.566 |
| | (18.388) | (17.691) | (12.726) |
| FF5 $\alpha$ | 0.065 | 0.125 | 0.151 |
| | (3.899) | (6.493) | (7.858) |
| SR($f$) | 0.719 | 0.719 | 0.719 |
| SR($f^{\sigma}$) | 0.833 | 1.141 | 1.289 |
| z($\Delta$SR) | 1.568 | 3.511 | 4.309 |

This table shows results for volatility managed multi-factor portfolios using the first 5 principal components (PC) of 137 monthly factor returns in Chen and Zimmermann (2022). They explain about 68 percent of the total variability. The weights at time $t$, $\boldsymbol{w}_t$ is given by $\boldsymbol{w}_t = \boldsymbol{\mu} + \boldsymbol{\theta}^\top \boldsymbol{\Phi}_t$. $\boldsymbol{\mu}$ represents the vector of passive, non-managed weights for each factor, $\boldsymbol{\theta}$ are coefficients to be estimated representing the sensitivities of each factor weight to the relevant volatility measures, and $\boldsymbol{\Phi}_t$ represents the observed volatility measures at time $t$. We consider both the annualized standard deviation of daily returns for individual factor returns during the preceding month, denoted by $\sigma_{i,t}$, as well as the implied volatility reported by the CBOE for the S&P 500 on the final trading day of the preceding month, denoted as $VIX_t$. $\theta_{\sigma,i}$ and $\theta_{VIX,i}$ are the vector of sensitivities of each factor to the relevant volatility measure. The first two columns report univariate estimations, where either $\theta_{\sigma,i}$ or $\theta_{VIX,i}$ is set to zero, respectively. The last column shows the multivariate portfolio. The first panel shows passive, non-managed weights for each individual factor ($\mu$), followed by optimal sensitivities of each factor to the relevant volatility measure ($\theta$), respectively. The volatility managed multi-factor portfolios have the same unconditional standard deviation as an optimal unmanaged multi-factor portfolio. The optimal unconditional mean-variance factor portfolio's weights are 1.532, −0.138, −0.153, −0.151, −0.241 and 0.000, for MKT, MKT, PC1, PC2, PC3, PC4, and PC5, respectively. The next part shows results from time-series regressions of each volatility managed multi-factor portfolio on the unconditional factor portfolio: $f_t^{\sigma} = \alpha + \beta f_t + \epsilon_t$ and Fama and French (2015) 5-factor $\alpha$'s. T-statistics are adjusted for heteroscedasticity and reported in brackets. The last part shows Sharpe ratios for the unmanaged (SR($f$)) and managed portfolios (SR($f^{\sigma}$)) and the $z$-statistic from the Jobson and Korkie (1981) test of the null that SR($f^{\sigma}$) − SR($f$) = 0. Alphas and Sharpe ratios are annualized. The sample period is July 1967 to December 2020.

## A.5 Volatility managed multi-factor portfolios with CRRA utility function

**Table A.11** Volatility managed multi-factor portfolios with CRRA utility function.

| | $\sigma_{i,t}$ | $VIX_t$ | $\sigma_{i,t}\&VIX_t$ |
|---|---|---|---|
| $\mu_{MKT}$ | 0.794 | 0.785 | 0.802 |
| $\mu_{SMB}$ | −0.251 | 0.647 | 0.145 |
| $\mu_{HML}$ | 1.732 | 1.773 | 2.053 |
| $\mu_{CMA}$ | 1.060 | 2.038 | 1.243 |
| $\mu_{RMW}$ | 1.821 | 3.262 | 2.582 |
| $\mu_{UMD}$ | 1.543 | 1.011 | 1.373 |
| $\theta_{\sigma,MKT}$ | −0.603 | | −0.761 |
| $\theta_{\sigma,SMB}$ | 8.422 | | 5.696 |
| $\theta_{\sigma,HML}$ | −20.992 | | −12.755 |
| $\theta_{\sigma,CMA}$ | 17.314 | | 15.072 |
| $\theta_{\sigma,RMW}$ | 2.744 | | 4.462 |
| $\theta_{\sigma,UMD}$ | −8.088 | | −7.657 |
| $\theta_{VIX,MKT}$ | | 1.063 | 0.883 |
| $\theta_{VIX,SMB}$ | | −0.020 | −0.531 |
| $\theta_{VIX,HML}$ | | −13.217 | −7.869 |
| $\theta_{VIX,CMA}$ | | 1.400 | −1.465 |
| $\theta_{VIX,RMW}$ | | −2.829 | −3.414 |
| $\theta_{VIX,UMD}$ | | −2.669 | 0.634 |
| $\alpha$ | 0.147 | 0.160 | 0.203 |
| | (4.591) | (6.350) | (6.245) |
| $\beta$ | 0.622 | 0.741 | 0.582 |
| | (10.416) | (17.103) | (10.435) |
| FF5 $\alpha$ | 0.173 | 0.203 | 0.226 |
| | (6.975) | (7.921) | (8.429) |
| SR($f$) | 1.237 | 1.237 | 1.237 |
| SR($f^{\sigma}$) | 1.416 | 1.622 | 1.614 |
| z($\Delta$SR) | 1.423 | 3.635 | 2.825 |

This table shows results for volatility managed multi-factor portfolios using a CRRA utility function, where $u(r_{p,t+1}) = (1 + r_{p,t+1})^{1-\gamma}/(1-\gamma)$, and the vector of factor weights at time $t$, $\boldsymbol{w}_t$ is given by $\boldsymbol{w}_t = \boldsymbol{\mu} + \boldsymbol{\theta}^{\top}\boldsymbol{\Phi}_t$. $\boldsymbol{\mu}$ represents the vector of passive, non-managed weights for each factor, $\boldsymbol{\theta}$ are coefficients to be estimated representing the sensitivities of each factor weight to the relevant volatility measures, and $\boldsymbol{\Phi}_t$ represents the observed volatility measures at time $t$. We consider both the annualized standard deviation of daily returns for individual factor returns during the preceding month, denoted by $\sigma_{i,t}$, as well as the implied volatility reported by the CBOE for the S&P 500 on the final trading day of the preceding month, denoted as $VIX_t$. $\theta_{\sigma,i}$ and $\theta_{VIX,i}$ are the vector of sensitivities of each factor to the relevant volatility measure. The first two columns report univariate estimations, where either $\theta_{\sigma,i}$ or $\theta_{VIX,i}$ is set to zero, respectively. The last column shows the multivariate portfolio. The first panel shows passive, non-managed weights for each individual factor ($\mu$), followed by optimal sensitivities of each factor to the relevant volatility measure ($\theta$), respectively. The volatility managed multi-factor portfolios have the same unconditional standard deviation as an optimal unmanaged multi-factor portfolio. The optimal weights for the unconditional mean-variance factor portfolio are 1.026, 0.498, −0.300, 3.263, 2.138, 0.699 for MKT, SMB, HML, CMA, RMW, and UMD, respectively. The next part shows results from time-series regressions of each volatility managed multi-factor portfolio on the unconditional factor portfolio: $f_t^{\sigma} = \alpha + \beta f_t + \epsilon_t$ and Fama and French (2015) 5-factor $\alpha$'s. T-statistics are adjusted for heteroscedasticity and reported in brackets. The last part shows Sharpe ratios for the unmanaged (SR($f$)) and managed portfolios (SR($f^{\sigma}$)) and the $z$-statistic from the Jobson and Korkie (1981) test of the null that SR($f^{\sigma}$) − SR($f$) = 0. Alphas and Sharpe ratios are annualized. The sample period is July 1967 to December 2020.

## Endnotes

1 Our work also relates to the broader literature on factor timing, which explores other, non-volatility related predictive variables. This literature includes Asness *et al.* (2000), Cohen *et al.* (2003), Kelly and Pruitt (2013), Garcia-Feijóo *et al.* (2015), Asness (2016), Asness *et al.* (2017), Hodges *et al.* (2017), Lee (2017), Dichtl *et al.* (2019), Gupta and Kelly (2019), Ilmanen *et al.* (2021), Haddad *et al.* (2020), Yara *et al.* (2021), Ehsani and Linnainmaa (2022), Neuhierl *et al.* (2023).

2 Contributions to this literature include Ang *et al.* (2006), Frazzini and Pedersen (2014), Ang *et al.* (2017) and Schneider *et al.* (2020).

3 Note that the expected return of such a portfolio depends linearly on the exposure, while the level of exposure has no effect on a portfolio's Sharpe ratio.

4 In the Supplementary Material, we also provide results for an objective function based on CRRA utility, which naturally captures effects of higher return moments. The findings align qualitatively with those reported in this section for the mean-variance setting.

5 http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

6 https://www.cboe.com/us/indices/dashboard/skew/.

7 We hereby emphasize that the CBOE skewness index is defined in a manner such that a higher index score indicates a more left-skewed distribution (and conversely, a less right-skewed distribution) of returns. For ease of interpretation, we will refer to regimes in which the CBOE skewness index is above the median as the "left-skewed" risk regime and to regimes in which the index is below the median, as the "right-skewed" regime.

8 Notably, the CRSP universe includes micro-cap stocks, which may be expensive to trade. Furthermore, factor portfolios require short positions, that can also be expensive to establish, at least for certain stocks. Taking these frictions into account could lead to different factor portfolio definitions, possibly based on a subset of fewer and more liquid stocks only, as compared to the original definition of the factors.

9 In unreported results, we use the volatility of last past month's daily market returns as a conditioning variable and find results very similar to those reported by DeMiguel *et al.* (2024).

10 This value of risk aversion is also chosen by several prominent related papers, for example, Brandt *et al.* (2009), Cederburg *et al.* (2020), DeMiguel *et al.* (2024).

11 Note that the level of alpha is a linear function of the portfolio weights, whereas $t$-statistics and Sharpe ratios are not.

12 We acknowledge that investors were unable to discern between regimes in real-time. Nevertheless, we observe that, a reasonably accurate median cut-off for regimes can be established within a few months. Subsequently, in Section 3.1, we perform an out-of-sample calculation for this threshold.

13 In Table A.1 in the Supplementary Material we report the results from unconditional double sorts on implied volatility and implied skewness. We find that the smallest performance improvements obtain in low implied volatility and left-skewed option-implied market regimes. Conversely, when implied volatility is high and index options indicate right-skewness, volatility management leads by far to the largest performance improvements. Compared to an unmanaged portfolio, volatility management can enhance the Sharpe ratio by 70%, to 1.917.

14 Results for univariate multi-factor portfolios, where either only past factor volatilities or the VIX is employed as a conditioning variable, are presented in the Supplementary Material.

15 Results for univariate multi-factor portfolios, utilizing only past factor volatilities or solely the VIX as conditioning variables, are provided in the Supplementary Material.

16 According to the NBER, there was also a short, two month recession following the onset of COVID-19 in February 2020, where this pattern is not present.

17 In instances where bid or ask quotes are missing for a particular month, we substitute the bid-ask spread from the preceding month. If we do not have any bid-ask information for a stock in a given quarter, we assign the median transaction costs of firms within the same size decile.

18 The findings for timed multi-factor portfolios, employing either past factor volatilities or solely the VIX as conditioning variables, are detailed in the Supplementary Material.

19 In unreported results, we observe a substantial decline in average portfolio leverage and turnover.

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